

CBMS Panel: Improving Teacher Education

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Teachers need to know the math they teach – and more!

“Teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach.”

– Task force on Teachers and Teacher Preparation, National Mathematics Panel 2008.

Mathematics for Teaching

Expert mathematics teachers

- ▶ Know mathematics as a **Scholar**:
 - ▶ its major results, its history of ideas, its connections to pre-college mathematics
- ▶ Know mathematics as an **Educator**:
 - ▶ how mathematical ideas develop in learners, algebra and arithmetic, geometry, analysis.
- ▶ Know mathematics as a **Mathematician**:
 - ▶ have deep experience of the doing of mathematics
 - ▶ grappling with problems, becoming completely absorbed in mathematical activity for a sustained period of time
- ▶ Know mathematics as a **Teacher**:
 - ▶ mathematics that is specific to the teaching profession
 - ▶ the craft of task design and the “mining” of student ideas

Knowing Mathematics as a Mathematician

- ▶ Examining the whole mathematical enterprise
 - ▶ as a coherent body of knowledge
 - ▶ as a way of thinking and inquiring about the world we live in
- ▶ Having *deep experience of the doing of mathematics* – e.g.
 - ▶ grappling with problems
 - ▶ building intuition
 - ▶ developing theories
 - ▶ becoming completely absorbed in mathematical activity for a sustained period of time

Teaching as an apprenticeship to a life of exploration:

... we thought of teaching as an art of developing in the very young curiosity, a sense of adventure, and a sensitivity of perception, as well as the mastery of basic skills. ... the teacher's mastery of the art of teaching ... is certainly not an obvious consequence of familiarity with factual information. ... the teacher's performance reflected [his/her] whole intellectual outlook.

Arnold Ross

Overview of the Mathematical Practice Standard

- ▶ Attend to precision
- ▶ Construct viable arguments
- ▶ Make sense of complex problems and persevere in solving them
- ▶ Look for structure
- ▶ Look for and express regularity in repeated reasoning
- ▶ Make strategic decisions about the use of technological tools

Experience first:

It has been observed in every human activity experience comes first, and as this experience grows the need for communication motivates the development of language. Sadly enough, in our classroom practice we place language first and experience second. We worry about what we should say in order to help the student understand. By this we mean to provide the effect of experience through the use of suitably chosen words. Not unexpectedly, the effect is at best a very pale image of the real thing.

Arnold Ross

Authentic Mathematical Experience

- ▶ Teachers and mathematicians experiencing mathematics together
 - ▶ as a collaborative activity
 - ▶ alongside students
 - ▶ as an empirical science
 - ▶ as exploration
- ▶ Key Features
 - ▶ emphasis on learning and problem-solving
 - ▶ depth over breadth
 - ▶ strengthening *mathematical habits of mind*

Two types of Mathematical Experiences

- ▶ Immersion Experiences
 - ▶ *Deeply personal* engagement in mathematical ideas
 - ▶ Extended experience of struggling with mathematics
 - ▶ Recommended as foundational course (not capstone)

- ▶ School-based mathematical activities
 - ▶ Study Groups
 - ▶ Teacher Math Circles

Typical features of an immersion experience

- ▶ Immersion in new ideas
- ▶ Necessity for open communication
- ▶ Acquiring taste for hard problems
- ▶ The central role of experience
 - ▶ empirical basis of mathematical knowledge
 - ▶ personal experience as guide for exploration
- ▶ Learning good judgement in recognizing significant ideas
- ▶ Sharing ideas with others
 - ▶ in writing
 - ▶ in seminars
- ▶ Questioning answers
- ▶ Low threshold – high ceiling

Variations on the immersion theme

- ▶ PROMYS for Teachers (Boston)
- ▶ Park City Mathematics Institute
- ▶ Arizona Teacher Institutes
- ▶ Rice University Mathematics Leadership Institute
- ▶ Math in the Middle Institute Partnership (Lincoln, Nebraska)
- ▶ Edutron – immersion institutes for elementary teachers (Massachusetts)
- ▶ Immersion institutes in the other sciences

Goals of Study Groups

Designed to support and sustain

- ▶ Ongoing mathematical activity among teachers (and mathematicians)
- ▶ School-based intellectual leadership in mathematics
- ▶ Learning cultures in school settings involving
 - ▶ Students
 - ▶ Teachers
 - ▶ Educators
 - ▶ Mathematicians

Typical features of Study Groups

- ▶ School-based professional learning communities.
- ▶ Collaboration of teachers with mathematicians.
- ▶ Teachers and mathematicians share interesting problems and solutions.

One teacher's description:

- ▶ First, we have fun doing math together.
- ▶ Sometimes we work on problems that connect to the math we teach in our classes.
- ▶ Sometimes we just work on what seems interesting.
- ▶ We always learn something.
- ▶ It makes us think about ways to give our students similar experiences.

Variations on the study group theme

- ▶ The *Focus on Mathematics* study groups (Boston)
- ▶ PROMYS academic year workshops
- ▶ PCMI Professional Development Outreach (PDO) groups
- ▶ Math Circles
 - ▶ rich in variations
 - ▶ sometimes school-based
 - ▶ for teachers
 - ▶ for students
 - ▶ for teachers and students

Mathematical Habits of Mind

- ▶ **Acquiring experience**
 - ▶ numerical experimentation and alert observation
 - ▶ mathematics as an empirical science
 - ▶ practice – enhancing skills
 - ▶ inductive reasoning – building intuition and sense-making
- ▶ **Use of language**
 - ▶ precision
 - ▶ asking good questions, formulating conjectures
 - ▶ reasoning – proofs and disproofs
- ▶ **Review**
 - ▶ identifying important ideas
 - ▶ making sense of complex problems
 - ▶ looking for connections
- ▶ **Generalization**
 - ▶ broadening applicability
 - ▶ questioning answers

Beliefs about the nature of mathematics

- ▶ **Mathematics is natural**
 - ▶ The empirical nature of mathematics
 - ▶ People do mathematics naturally
- ▶ **Mathematics exists independent of us**
 - ▶ We can perform experiments
 - ▶ We can test ideas and decide for ourselves
- ▶ **Experience precedes formality**
 - ▶ “Meaning” is determined by experience
 - ▶ Definitions and theorems are capstones
 - ▶ Language is a tool for coming to terms with experience
- ▶ **Mathematics is the study of structure**
 - ▶ Operations, order
 - ▶ Shape
 - ▶ Continuity
 - ▶ Transformation
- ▶ **Mathematics is the art of figuring things out**

On Teaching and Teacher Preparation

We teach mathematics the way we learn(ed) mathematics

- ▶ “Give a man a fish, and he’ll eat for a day. Teach a man *how to fish* and he’ll eat for a lifetime.” – Lao Tzu (6th century BC)
- ▶ “To every thing there is a season, and a time to every purpose under the heaven” – Ecclesiastes 3:1.