Mathematical Practice, Elementary Grades

The work of teaching mathematical practice

This section provides brief sketches of the Common Core State Standards for Mathematical Practice as they apply to teaching in elementary school.

1 Make sense of problems and persevere in solving them. Young children are eager for challenges and are problem solvers by nature. A challenge for elementary teachers is to help children maintain their enjoyment for engaging with problems. Teachers can help their students explore, investigate, and persevere in solving problems by creating a nurturing classroom environment. It is important for teachers to convey that everyone can learn math and that it takes active effort and thinking to do so. It is also important for teachers to convey that by thinking hard, we can actually increase our intelligence. Research on motivation indicates that supporting autonomy, competence, and relatedness supports internal motivation and leads to better outcomes than environments that are experienced as highly controlling. Elementary school teachers often want to make mathematics "fun" for students and shelter them from the difficulty of learning mathematics, which frequently leads to activities that have little mathematical substance.

2 Reason abstractly and quantitatively. Much of elementary teachers' work is helping young students connect their observations about quantities in the world with the abstract mathematical symbols we use to describe, record, and reason about relationships among quantities. Objects or simple math drawings (e.g., drawings that show tens and ones) can be especially helpful in making such connections, so elementary teachers need to be well prepared to use and discuss such drawings or objects in their instruction.

3 Construct viable arguments and critique the reasoning of others. Mathematics is about ideas, so mathematical arguments and lines of reasoning are an important part of mathematics, even at elementary school. For example, elementary school students need to give arguments for why strategies for adding, subtracting, multiplying, and dividing whole numbers work, and similarly with fractions. Students cannot make sense of mathematical arguments unless they think actively about them, which includes making their own arguments and carefully listening to and evaluating other people's arguments.

4 Model with mathematics. In elementary school, modeling with mathematics often involves writing an equation for a situation and then solving the equation to solve a problem about the situation. Students also model with mathematics when they draw a quadrilateral to show a route that started and ended at the same location and had four turns. At elementary school, modeling with mathematics is often *mathematizing*—which means focusing on the mathematical aspects of a situation and formulating it in mathematical terms. For example, students may notice shapes in objects around them, such as triangular bracing in chairs or

quadrilaterals in collapsible gates. Teachers also need to help students notice math in the world around them.

5 Use appropriate tools strategically. Math drawings, such as drawings of tens and ones or hundreds, tens, and ones, can be an especially valuable tool in elementary school. Students use math drawings strategically when they use them to make sense of numerical work and not just for rote calculations. Elementary school students also need to learn to use tools such as rulers and protractors. Teachers should know how to guide students to make strategic use of these tools. Technology tools can also be used effectively in elementary schools, and teachers need to think carefully about how and when to use such tools. A common assertion in elementary schools is that children should not use calculators until they have memorized all of the number combinations for the four operations. However, elementary school students can use calculators effectively for problem solving to tackle mathematics for which they understand the operation but do not yet have facility with the computation. For example, children who understand the concept of multiplication but who have not yet mastered a means for multiplying multi-digit numbers can use calculators to determine how many times their heart beats in a day, week, month, or year after counting the number of beats in a minute. Thus, teachers need to have opportunities to think carefully about using technology tools strategically.

6 Attend to precision. Elementary school students attend to precision when they take care to make math drawings and carefully coordinate them with numerical work, such as when they show how to decompose a rectangle into component parts that correspond to the partial products in a multiplication problem. They also attend to precision when they describe a line of reasoning with care, attending to the key points and choosing their words to say exactly what they mean.

7 Look for and make use of structure. At elementary school, looking for structure usually involves the following:

- unitizing—finding or creating a unit, such as seeing one hundred as ten groups of ten, viewing a unit fraction such as 1/5 as a new unit, or making a car shape out of several smaller shapes and then repeating the car shape to show traffic on a road*;*
- decomposing and composing, such as decomposing 1 ten into 10 ones or viewing a rectangular prism as composed of equal layers made of unit cubes;
- relating and ordering, such as putting a collection of sticks in order by length or reasoning that 1/11 is greater than 1/12 because when a cake is cut into 12 equal pieces the pieces are smaller than when it is only cut into 11 equal pieces; and
- looking for patterns and structures and organizing information, such as noticing the repeating pattern of ones digits in the multiples of a number or that when you make quadrilaterals out of sticks so that the opposite sides are the same length, the opposite sides are also always parallel.

8 Look for and express regularity in repeated reasoning. Elementary school students require repeated opportunities to reason about and make sense of strategies and methods. It is through repeated reasoning that students will be able to make sense of calculation methods and use them with understanding.

Key understandings to support this work

Preparation for teaching the CCSM mathematical practice standards should include regular opportunities to engage deliberately in mathematical practices. Almost all teachers whose reflect the work described in the previous section have had substantial opportunities to experience learning in similar environments. In particular, it is important for teachers to experience the enjoyment and satisfaction of working hard at solving a problem so that they realize that this sort of intellectual work can also be "fun" (albeit in a different sort of way than more "playful" activities are fun). Key understandings for teaching mathematical practice in the elementary grades can be achieved if teachers have learning experiences such as:

- Solving challenging problems
- Engagement in mathematical arguments and lines of reasoning
- Active listening to and evaluating other people's arguments
- Being precise and deliberate when discussing one's own reasoning
- Understanding the importance of careful definition and learning to use mathematical terminology correctly
- Reflecting on and looking for examples of unitizing, decomposing and composing, relating and ordering
- Looking for patterns and structures and organizing information
- Generalizing from examples

Illustrative Examples

The following are sample problems that can provide elementary teachers with experiences such as those described in the previous section.

Which whole numbers can be expressed as the difference of two perfect squares? Provide an explanation that justifies your answer.

There's a famous fast-food restaurant where you can order chicken nuggets in boxes of 6, 9 and 20. By combining boxes of one or more size, there are many orders you can place. For example, if you order a box of 20 and two boxes of 6, you get 32 chicken nuggets. What is the largest number of chicken nuggets that you cannot order exactly? Explain why you can't order that number but can order a combination equal to any larger number.

A cube with edges of length 2 centimeters is built from centimeter cubes. If you paint the faces of this cube and then break it into centimeter cubes, how many cubes will be painted on three faces? How many will be painted on two faces? On one

face? How many will be unpainted? What if the edge has a length different from 2? What if the edge length of the large cube is 3, 4 or 5 cm? What if the edge length of the cube was 20 cm? Can you generalize this to an edge length of *n* cm?

Heather went to the store to purchase ink pens. She found three kinds of pens. The first cost $4 each; the second was 4 for $1; and the cost for the third kind was 2 for $1. She bought 20 pens and she bought at least one of each kind. The cost was $20. How many of each kind did I buy? Provide an explanation that shows how you solved the problem. Is there more than one answer?