Ratio and Proportional Relationships, Grades 6 – 7

The work of teaching Ratio and Proportional Relationships

Ratios and proportional relationships are used to describe how quantities are related and how quantities vary together. Their study bridges multiplication and division of the elementary grades with linear functions in the upper middle grades and high school.

Teachers may introduce ratios with juice or paint mixtures and walking distances and times, for example. When two juices or paints are mixed, there are certain quantities that yield mixtures that have identical flavors or colors. When people walk, there are distances and times that correspond with the same pace of walking. Such situations motivate the concepts of ratio and rate.

Teachers teach students to represent ratios in tables, on double number lines, and with tape diagrams and to use ratio language (e.g., "A for every B," "A parts to B parts") in discussing ratios. With the aid of these representations, students learn to reason about quantities that are in equivalent ratios and they solve ratio problems flexibly, and in increasingly sophisticated and abbreviated ways.

A central idea in the study of ratio and proportional relationships is the notion of a (unit) rate, which tells the amount of one quantity per 1 unit of another quantity. Teachers emphasize this rate language and they guide students toward problem-solving methods that rely on unit rates. As students work with proportional relationships, in which two quantities vary together, teachers help students see that a constant of proportionality, c, in an equation of the form $y=cx$ and the slope of the graph are unit rates.

Especially important is that teachers guide students to understand the distinction between additive relationships and proportional relationships, which are commonly confused. Teachers must draw students' attention to wording such as "for every," "for each," and "per," which indicate a proportional relationship. They may also highlight that "for each" and "per" refer implicitly to 1 unit of a quantity. For example, 3/2 meters per second means 3/2 meters for every 1 second.

Ratios and proportional relationships are widely applicable. Teachers will teach students to use them in geometry, when working with scale drawings, and in statistics, when using samples to make inferences about populations. They will also teach multi-step problems in which ratios are used, as in situations of percent increase or decrease.

Key understandings to support this work

- Know the meanings of ratio and rate and use ratio and rate language.
- Explain how to use tables, double number lines, and tape diagrams to represent and reason about ratios and equivalent ratios and to solve problems.
- Find unit rates in tables and on double number lines and use unit rates to describe situations and solve problems.
- Represent proportional relationships with tables, graphs, and equations and explain how to relate the different representations.
- Know how to determine when quantities described in a situation or with a table or graph are or are not in a proportional relationship.
- Recognize that a common error is for students to make an additive comparison in cases where such a comparison does not apply.
- Know ways of reasoning about and solving simple percent problems as well as multi-step percent problems, such as those involving percent increase and percent decrease.

Illustrative examples

Will a mixture of 1 cup blue paint with 3 cups yellow paint be the same color as a mixture of 4 cups blue paint and 6 cups yellow paint? Why might sixth graders say that these two paint mixtures will be the same color? Make two ratio tables, one for each paint mixture, and use these tables to explain in several ways how the mixtures compare.

To make "grapple juice," we mix grape juice and apple juice in a ratio of 2 to 3. Describe ways to interpret each of the fractions 2/3, 3/2, 2/5, and 3/5 in terms of grapple juice (*other than* as ratios). [e.g., There is 2/3 of a cup of grape juice in the mixture for every 1 cup of apple juice. There is 2/3 times as much grape juice in the mixture as apple juice.] Show how these fractions occur in tables, equations, and graphs relating quantities of juice.

If it takes 4 people 6 hours to mow a field, how long will it take 2 people to mow a field of the same size? (Assume all the people work at the same steady rate.) Can we solve this problem by setting up and solving a proportion, 4 people/6 hours = 2 people/H hours? Why or why not? How long would it take 3 people to mow a field of the same size? Is that amount of time half way between the amounts of time for 4 people and 2 people or not?

Some pants are on sale for 20% off. The new, reduced price is $30. Why can't you find the original price by calculating 20% of $30 and then adding that to $30? How can you reason correctly to find the original price?