What do teachers need to know to teach mathematics well?

10 minute presentation at CBMS

October 11, 2010

In the 1960's, there was the **New Math**. In the 2010's, we have the **Common Core Standards.**

The New Math failed miserably, mainly because:

1. The mathematics imposed on the schools was too often out of touch with reality.

2. *The teachers did not have the requisite content knowledge to implement the new program.*

3. Textbooks did not provide teachers with the needed support.

We are on the threshold of entering the Common Core era. Common Core may yet fail, because:

1. The mathematical topics in the Common Core Standards are what they have always been, but they have been re-organized to avoid the time-honored mathematical errors.

2. *Many teachers may not have the requisite content knowledge to implement the new program.*
Having seen things done the wrong way too many times, they may not recognize what is right anymore.

3. There is no indication that textbooks will do any better than last time.

Many hurdles lie ahead still, but **the one about teachers' content knowledge is the most critical.**

If teachers have to teach what they don't know, then no amount of improvement in other areas will make any difference:

not textbooks,

not pedagogical practices,

not teaching tools,

not equity issues,

not psychology of learning,

not administrative support, etc.

The blame for teachers' lack of content knowledge must be shared by one and all. Educators, mathematicians, policy makers, teachers themselves . . .

This topic is worthy of a one-week symposium all by itself.

In March, I made a short presentation at a university about the Common Core Standards based on the March draft. I made a passing reference to the most mundane of topics: how to teach fraction multiplication in grade 5.

In my PPT, I *outlined* four main steps in the Common Core approach to this topic, as follows:

• A fraction is defined as a certain point on the number line. One may therefore identify a fraction with the length of a segment.

• The product $5 \times \frac{2}{3}$ is the length of 5 copies of (a segment of length) $\frac{2}{3}$.

• The product $\frac{1}{4} \times \frac{2}{3}$ is the length of 1 part when a segment of length $\frac{2}{3}$ is divided into 4 equal parts.

• Finally, $\frac{5}{4} \times \frac{2}{3}$ is defined to be the length of 5 copies of $\frac{1}{4} \times \frac{2}{3}$, leading to the proof of the formula $\frac{5}{4} \times \frac{2}{3} = \frac{5 \times 2}{4 \times 3}$.

An elementary teacher asked afterwards how she could be expected to implement these standards.

Almost her exact words: "I have just talked to the other teachers, and none of us can teach this way."

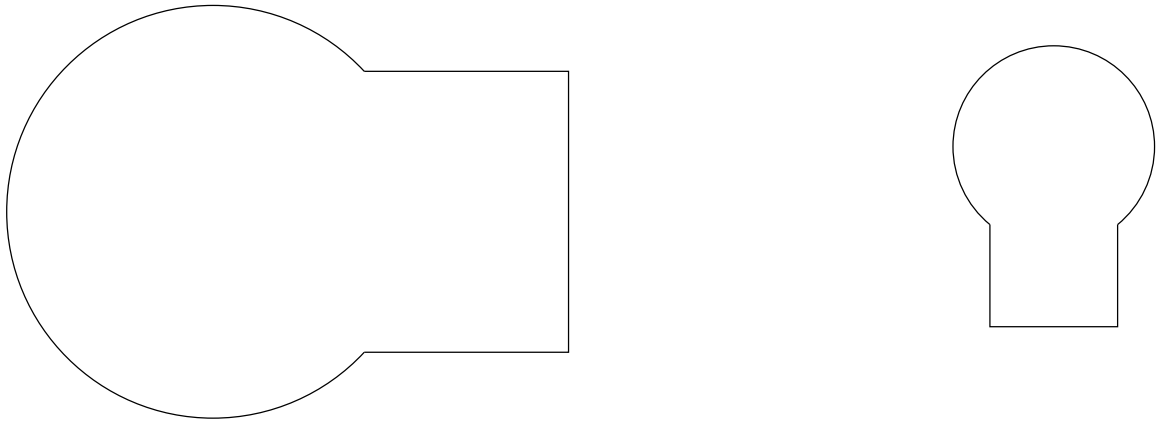
In the last months, I have received quite a few inquiries about the Common Core. Let me quote two teachers.

(1) [About the introduction of rigid motions and dilation to define congruence and similarity]

"If you don't understand the proportionality of similarity what good is it? Who cares what flips, rotations, or translations it took to get from one figure to another..... Where's the math here?

"I have never used these ideas in any mathematics I have ever used either in the classroom or in 17 years of construction.... It is a waste of classroom time in my mind."

Brief explanation: How to explain that the following two figures are similar?



The school curriculum uses "congruence" and "similarity" without ever saying what they mean. It tries to make believe that "*same size and same shape*" and "*same shape but not necessarily the same size*" are mathematical definitions, but they are not. Common Core tries to correct this error.

(2) [About the introduction of similar triangles for the study of linear equation of two variables]

"After 13 years of teaching high school algebra, I wonder why you see similarity as critically important to algebra I mastery — that certainly never occurred to me as a teacher of algebra. . . . What makes you say that a student needs to understand similar triangles in order to write the equation of a straight line between two points?"

Without similar triangles, one cannot make sense of the **slope** of a line. (*Education research into why students don't understand slope should begin with this error in the school curriculum.*)

Consequently, students don't know why the graph of a linear equation is a straight line. Everything about the graph of linear equations of two variables therefore must be learned by rote.

Certain entrenched errors in the school curriculum have become the norm.

Any attempt to rectify the errors is now looked upon with suspicion.

Until we can get teachers to know correct mathematics, **and be comfortable teaching correct mathematics in the school classroom,** the Common Core Standards won't go very far.

Should the Common Core Standards fail, it will **not** be primarily because teachers:

do not use correct classroom strategies,

do not focus on the learners,

cannot communicate with students,

are not sensitive to students' thinking,

do not know how to build a norm for mathematical discourse,

do not have access to the right kind of tasks,

do not have the expertise to refine teaching practices,

etc., etc.

All these will play some role, no doubt, as they always do under any circumstance.

But if the **New Math** is any guide, the Common Core Standards would fail principally because **teachers don't know the mathematics in the Common Core Standards.**