

Number and Operations – Fractions, Grades 3 - 5

The work of teaching number and operations – fractions

These standards concern the meaning of fractions, comparing fractions, and calculating with fractions. Fractions are well known to be difficult to teach and learn, but the study of fractions is an important springboard to algebra. It is therefore especially important for grades 3 - 5 teachers to have solid conceptions of fractions as numbers, to know the progression for teaching fractions described in the standards and to be able to help students represent fractions, use representations to reason about fractions, and explain the logic underlying equivalent fractions, comparing fractions, and calculating with fractions.

Teachers help students understand that fractions are numbers, that unit fractions are the basic unit for fractions, and that every fraction is a sum of unit fractions. Students learn that the operations of addition, subtraction, multiplication, and division retain their meanings from whole numbers, although extending these meanings to the case of fractions is not automatic or easy for students. Students learn that multiplication and division with fractions less than one reverses the size relationships students are used to with whole numbers (e.g., multiplying a number by a whole number yields a larger product but multiplying a number by a fraction less than one yields a smaller product). Teachers must be aware that the connection between fractions and division requires investigation and discussion. For example, the fraction $\frac{3}{4}$ is initially defined as a sum of three $\frac{1}{4}$ and then as 3 times $\frac{1}{4}$, but 3 divided by 4 may be thought of as the amount in one share when 3 equal things are divided equally among 4 shares. Only teachers with a good understanding of fractions can facilitate the reasoning that is required to connect this situation of 3 divided by 4 with the fraction $\frac{3}{4}$.

The use of number lines to represent fractions requires special attention by teachers because of difficulties that are known to occur with this representation. In particular, teachers must know the importance of drawing attention to intervals on the number line because of the tendency for students to attend only to tick marks and not to lengths of intervals.

Key skills and understandings needed to begin teaching this domain

- Understand that fractions are numbers, that unit fractions are the basic units for fractions and be able to use number lines to represent fractions.
- Know how to represent fractions and to use representations to reason about fractions and to facilitate discussions about equivalent fractions.
- Know several approaches to comparing fractions and be able to explain why “cross multiplying” works as a way of comparing fractions.
- Understand how to convert between fractions, decimal fractions and percents and know which fractions correspond to finite decimals.
- Know how to add, subtract, multiply and divide fractions and realize that the basic rules of arithmetic (e.g. the distributive property) holds true for fractions.

Illustrative Examples

Use the meaning of fractions to explain why $\frac{6}{8} = \frac{9}{12}$, providing an explanation that is appropriate for an elementary classroom in which students are first learning about equivalent fractions.

Find two fractions between $\frac{8}{13}$ and $\frac{10}{17}$ by using two different approaches, neither of which involves using decimals and explain how you know your answer is between the two fractions.

Use good number sense (without using decimals) to place the fractions $\frac{9}{16}, \frac{4}{11}, \frac{4}{13}, \frac{5}{8}, \frac{5}{13}$ in order from smallest to largest and provide a justification that does not involve any use of a calculator.

Explain a process that can be used to write any fraction as the sum of distinct unit fractions and demonstrate the process by decomposing $\frac{6}{7}$ and $\frac{5}{21}$ as the sum of distinct unit fractions.