Chapter 2 footnotes with hyperlinks

The footnotes from Chapter 2 are listed below and hyperlinked (when possible) to the references cited.

Many of the documents cited are freely available. National Research Council reports such as *Adding It Up* can be read on-line. They can be downloaded without charge as can documents from the Conference Board of the Mathematical Sciences and the Council of Chief State School Officers. In some cases, cited portions of documents can be seen via Google Books.

Mathematics education research journal articles are likely to require a subscription. At many academic institutions, these journals will be accessible via institutional subscription. Attempts to access a JSTOR link without such a subscription will get the response "Cannot download the information you requested."

1. These were: *Einleitung zur Rechen-Kunst* (*Introduction to the Art of Reckoning*), St Petersburg (vol. 1, 1738, vol. 2, 1740); *The Elements of Arithmetic*, London, 1830.

2. Hodgson points out that "one could even see the ICMI as having been formed on the very assumption that university mathematicians should have an influence on school mathematics." See *The Teaching and Learning of Mathematics at University Level*, Kluwer, 2001, p. 503.

3. Murray discusses the polarization of teaching and research within the U.S. mathematical community in *Women Becoming Mathematicians: Creating a Professional Identity in Post–World War II America*, MIT Press, 2000, pp. 6–10. For examples of U.S. mathematician involvement (e.g., the founding of the International Commission on the Teaching of Mathematics (later ICMI) at the International Congress of Mathematicians) and social context of its diminution, see Donoghue, "The Emergence of a Profession: Mathematics Education in the United States, 1890–1920," in *A History of School Mathematics,* vol. 1, NCTM, 2003. Changes in twentieth-century psychology research were also a factor, see Roberts, "E. H. Moore's Early Twentieth-Century Program for Reform in Mathematics Education," *American Mathematical Monthly*, 2001.

4. *Teaching Teachers Mathematics* (Mathematical Sciences Research Institute, 2009) gives an overview of past and recent counterexamples.

5. In 2010, Masingila et al. surveyed 1,926 U.S. higher education institutions that prepared elementary teachers. Of those who responded (43%), less than half reported giving training or support for instructors of mathematics courses for elementary teachers. However, the authors write that "there appears to be interest in training and support as a number of survey respondents contacted us to ask where they could find resources for teaching these courses." See "Who Teaches Mathematics Content Courses for Prospective Elementary Teachers in the United States? Results of a National Survey," *Journal of Mathematics Teacher Education*, 2012.

6. Quoted from Schoenfeld, "Learning to Think Mathematically" in *Handbook for Research on Mathematics Teaching and Learning*, 1992, p. 359. Note that these beliefs may not be explicitly stated as survey or interview responses, but displayed as classroom behaviors, e.g., giving up

if a problem is not quickly solved. This discussion is not meant to exclude the possibility of exceptional mathematical talent, but focuses on the idea that K–12 mathematics can be learned in its absence.

7. Stevenson and Stigler documented similar beliefs among U.S. first and fifth graders, and their mothers, but found that their Japanese and Chinese counterparts focused more on effort rather than ability. See Chapter 5 of *The Learning Gap*, Simon & Schuster, 1992. See also *Data Compendium for the NAEP 1992 Mathematics Assessment for the Nation and the States*, National Center for Educational Statistics, 1993.

8. Note that such beliefs may vary according to domain, e.g., one may believe in a "math gene," but favor continued practice in order to improve sports performance.

9. For a brief overview of research in this area, including classroom studies, see Dweck, "Mind-sets and Equitable Education," *Principal Leadership*, 2010. For a review of research and recommendations for classroom practice, see *Encouraging Girls in Math and Science* (IES Practice Guide, NCER 2007-2003), Institute of Educational Sciences, 2007, pp. 11–13.

10. See Michalowicz & Howard, "An Analysis of Mathematics Texts from the Nineteenth Century" in *A History of School Mathematics*, vol. 1, NCTM, 2003, especially pp. 82–83.

11. Lambdin & Walcott, "Changes through the Years: Connections between Psychological Learning Theories and the School Mathematics Curriculum," *The Learning of Mathematics*, 69th Yearbook, NCTM, 2007. For discussion of current practices, see Ma, "Three Approaches to One-Place Addition and Subtraction: Counting Strategies, Memorized Facts, and Thinking Tools," unpublished.

12. This is a slight reformulation of Lampert, 1990 as quoted by Schoenfeld, "Learning to Think Mathematically" in *Handbook for Research on Mathematics Teaching and Learning,* 1992, p. 359. The surrounding text discusses research on school experiences that shape such beliefs.

13. For example, see Hiebert et al.'s study of eighth grade classrooms, *Teaching Mathematics in Seven Countries: Results from the TIMSS 1999 Video Study*, U.S. Department of Education, 2003.

14. See analyses of data from the TIMSS video studies of 1999 (Hiebert et al., pp. 73–75) and of 1995 (Manaster, *American Mathematical Monthly*, 1998).

15. Schmidt and Houang analyzed the content and sequencing of topics in grades 1–8 in the U.S. and other countries. See "Lack of Focus in the Mathematics Curriculum," in *Lessons Learned*, Brookings Institution Press, 2007, p. 66. Examples of treatments of fractions and negative

numbers that do not afford deductive reasoning are given by Wu in "Phoenix Rising," *American Educator*, 2011.

16. For example, middle grades and high school teachers who participated in an MSP based on an immersion approach (involving intensive sessions of doing mathematics) reported changes in beliefs that affected their teaching, e.g., communicating that it is "OK" to struggle. See ," *Focus on Mathematics Summative Evaluation Report 2009*, p. 73. Gains in student test scores are shown on p. 93 (high school) and p. 96 (middle grades).

17. For a snapshot from one such collaboration, see *Teaching Teachers Mathematics*, Mathematical Sciences Research Institute, 2009, p. 34; for descriptions of three Math Science Partnerships, see pp. 32–41.

18. Test quality can be a major limitation for this measure. An analysis of state mathematics tests found low levels of cognitive demand, e.g., questions that asked for recall or performance of simple algorithms, rather than complex reasoning over an extended period. See Hyde et al., "Gender Similarities Characterize Math Performance," *Science,* 2008, 494–495.

19. See *Preparing Teachers: Building Evidence for Sound Policy*, National Research Council, 2010, p. 112. See also, Telese, "Middle School Mathematics Teachers' Professional Development and Student Achievement," *Journal of Educational Research*, 2012. Telese's measure of student achievement was the Grade 8 National Assessment of Educational Progress, which includes items with a high level of cognitive demand. It found number of mathematics courses to be a strong predictor, but like many such studies, it did not have an experimental or quasi-experimental design.

20. Shulman, "Those Who Understand: Knowledge Growth in Teaching," *Educational Researcher*, 1986.

21. On average, the prospective secondary teachers had taken over 9 college-level mathematics courses. Ball, "Prospective Elementary and Secondary Teachers' Understanding of Division," *Journal for Research in Mathematics Education*, 1990.

22. Hill et al., "Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement," *American Educational Research Journal*, 2005.

23. Cohen & Hill, "Instructional Policy and Classroom Performance: The Mathematics Reform in California," *Teachers College Record*, 2000.

24. Blank & Atlas, Effects of Teacher Professional Development on Gains in Student Achievement,

Council of Chief State School Officers, 2009.

25. Perry & Lewis, *Improving the Mathematical Content Base of Lesson Study: Summary of Results*, 2011.

26. Example from Ma, *Knowing and Teaching Elementary Mathematics*, Erlbaum, 1999, p. 15. Similar examples occur in other East Asian countries. Lewis et al. describe how Japanese teacher's manuals may support teachers' perceptions of paths in "Using Japanese Curriculum Materials to Support Lesson Study Outside Japan: Toward Coherent Curriculum," *Educational Studies in Japan: International Yearbook*, 2011.

27. Sarama & Clements, *Early Childhood Mathematics Education Research*, Routledge, 2009, pp. 352–363.

28. See special issue on learning trajectories, *Mathematical Thinking and Learning*, 2004.

29. See Duncan et al., "School Readiness and Later Achievement," *Developmental Psychology*, 2007; Claessens et al., "Kindergarten Skills and Fifth-grade Achievement: Evidence from the ECLS-K," *Economics of Education Review*, 2009; Siegler et al., "Early Predictors of High School Mathematics Achievement," *Psychological Science*, 2012. These studies examined large longitudinal data sets from the U.S. and other countries.

30. *Preparing Teachers: Building Sound Evidence for Sound Policy*, National Research Council, 2010, pp. 114–115.

31. This is made explicit for early childhood educators in *Mathematics Learning in Early Childhood*, National Research Council, 2009, pp. 3–4.

32. Ingersoll & Merrill, "Who's Teaching Our Children?," *Educational Leadership*, 2010.

33. Ingersoll & Perda, "Is the Supply of Mathematics and Science Teachers Sufficient?," *American Educational Research Journal*, 2010.

34. Ingersoll & May, "The Magnitude, Destinations, and Determinants of Mathematics and Science Teacher Turnover," Consortium for Policy Research in Education, 2010, pp. 44, 46.

35. *Preparing Teachers: Building Sound Evidence for Sound Policy*, National Research Council, 2010, pp. 34–39.

36. Ingersoll & Merrill, "Retaining Teachers: How Preparation Matters," *Educational Leadership*, 2012. See also Darling-Hammond, *Solving the Dilemmas of Teacher Supply, Demand, and Standards*, National Commission on Teaching and America's Future, 2000, pp. 17–19; *Tenth Anniversary Report*, UTeach, 2010, p. 16.

37. CAEP was formed by the merger of the National Council for the Accreditation of Teacher Education (NCATE) and the Teacher Education Accreditation Council (TEAC). Two of the MET II writers are engaged in the development of the CAEP standards.

38. *Key State Education Policies on PK–12 Education: 2008*, Council of Chief State School Officers, p. 22.

39. CBMS 2005 Survey, Table SP.3.

40. *National Impact Report: Math and Science Partnership Program*, National Science Foundation, 2010, p. 15.

41. In addition to the forthcoming CAEP standards, note the 2012 report *Supporting Implementation of the Common Core State Standards for Mathematics: Recommendations for Professional Development*, Friday Institute for Educational Innovation at the North Carolina State University College of Education.

42. For an overview of MSP outcomes, including increases in student achievement, see *National Impact Report: Math and Science Partnership Program*, National Science Foundation, 2010, pp. 6, 10–12.