

CHAPTER 3

Recommendations: Mathematics for Teachers; Roles for Mathematicians

This document’s six recommendations are presented in two groups: the mathematics that teachers need to know; and mathematicians’ roles in the mathematical education of teachers.

A. Mathematics for Teachers

The term “teacher of mathematics” includes early childhood and elementary-level generalist teachers as well as middle grades and high school teachers who teach mathematics classes. It also includes teachers of special needs students, English Language Learners, and other special groups, when those teachers have direct responsibility for teaching mathematics.

These recommendations are intended to apply to any pathway for teacher preparation and credentialing, regardless of form and type of institution, including undergraduate and graduate degree and certification programs, as well as any alternative program that prepares teachers of mathematics. It is assumed that this required coursework in mathematics is complemented with appropriate coursework in education, especially courses in methods of teaching mathematics. Implicit in these recommendations is that mathematicians and statisticians should teach, or co-teach, the mathematics discussed in these recommendations and that this instruction should occur at accredited institutions of higher education.¹

Recommendation 1. Prospective teachers need mathematics courses that develop a solid understanding of the mathematics they will teach. The mathematical knowledge needed by teachers at all levels is substantial yet quite different from that required in other mathematical professions. Prospective teachers need to understand the fundamental principles that underlie school mathematics, so that they can teach it to diverse groups of students as a coherent, reasoned activity and communicate an appreciation of the elegance and power of the subject. Thus, coursework for prospective teachers should examine the mathematics they will teach in depth, from a teacher’s perspective.

Recommendation 2. Coursework that allows time to engage in reasoning, explaining, and making sense of the mathematics that prospective teachers will teach is

¹The recommendations for teacher preparation in this report are formulated in terms of courses and semester-hours, but this is not meant to exclude other ways of awarding credit or organizing teacher education. For example, collegiate institutions that do not follow a semester system with most courses earning 3 credit-hours will need to adapt these recommendations accordingly.

needed to produce well-started beginning teachers. Although the quality of mathematical preparation is more important than the quantity, the following recommendations are made for the amount of mathematics coursework for prospective teachers.

- i. *Prospective elementary teachers should be required to complete at least 12 semester-hours on fundamental ideas of elementary mathematics, their early childhood precursors, and middle school successors.*
- ii. *Prospective middle grades (5–8) teachers of mathematics should be required to complete at least 24 semester-hours of mathematics that includes at least 15 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.*
- iii. *Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics that includes three courses with a primary focus on high school mathematics from an advanced viewpoint.*

Recommendations for the content and nature of this coursework are outlined in Chapters 4, 5, and 6 of this report.

At each level, these recommendations include courses especially designed for teachers. At the middle grades and high school levels, these recommendations also include courses such as calculus, linear algebra, and history of mathematics, which are designed for and taken by a wider undergraduate audience. The recommended statistics–probability courses are different from the statistics courses recommended in MET I because they focus on the data collection, analysis, and interpretation needed to teach the statistics outlined in the CCSS. Such courses are likely to be different from the more theoretically-oriented probability and statistics courses typically taken by science, technology, engineering, and mathematics (STEM) majors, and from the non-calculus-based statistics courses offered at many universities.

In states where teacher certification is accomplished as part of a post-baccalaureate program, a mathematics-intensive undergraduate major along with a minor in mathematics for teaching would be an acceptable preparation for the graduate degree in mathematics teaching. This graduate degree should include mathematics courses with a primary focus on high school mathematics from an advanced standpoint.

Although elementary certification in most states is still a K–6 and, in some states, a K–8 certification, state education departments and accreditation associations are urged to require all grades 5–8 teachers of mathematics to satisfy the 24-hour requirement recommended by this report.

Elementary mathematics specialists play important roles in elementary teachers’ professional development. Those roles, and the education of specialists and early childhood teachers are discussed in Chapter 4.

Recommendation 3. Throughout their careers, teachers need opportunities for continued professional growth in their mathematical knowledge. Satisfying the minimum requirements for initial certification to teach mathematics does not ensure that even outstanding future teachers have the knowledge of mathematics, of teaching, and of students that is possessed by successful experienced teachers. Like all

professionals, teachers need opportunities for professional growth throughout their careers. This need, however, takes on increased importance due to the wide adoption of the CCSS.

In-service programs should offer teachers content-based professional growth at levels appropriate for their experience, as they make the transition from new teacher, to mid-career professional, to master teacher. Opportunities for mathematical growth should include school- and district-based professional development, university-based graduate courses and “short courses” (e.g., one- or two-week intensive courses), teacher-driven professional experiences (e.g., lesson study), and teacher–mathematician partnerships (e.g., math teachers’ circles).² There is an important role for mathematicians in all these activities.

Regular opportunities to learn mathematics beyond preparation courses are particularly important at the high school level. A reasonable goal for initial certification at this level is to create beginning teachers who are able to teach competently a portion of the high school curriculum and who are prepared to learn throughout their careers from their teaching and professional development experiences. Most well-prepared new high school teachers will be ready to teach algebra and geometry. But, most new high school teachers will require further coursework to be well prepared to teach subjects such as precalculus, calculus, discrete mathematics, matrix algebra, and more than basic statistics.

Recommendation 4. All courses and professional development experiences for mathematics teachers should develop the habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing. Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop these habits of mind in their students. This recommendation is at least as important for practicing teachers as future teachers. A worthy goal of mathematics instruction for any undergraduate is to develop not only knowledge of content but also the ability to work in ways characteristic of the discipline. For teachers, this is not only worthy, but necessary. In order to develop these abilities in their students, teachers must experience them in their own mathematical education, through, for example, immersion experiences, research projects, or seminars devoted to doing mathematics. To help their students achieve the CCSS Standards for Mathematical Practice, teachers must not only understand the practices of the discipline, but how these practices can occur in school mathematics and be acquired by students.

B. Roles for Mathematicians in Teacher Education

Recommendation 5. At institutions that prepare teachers or offer professional development, teacher education must be recognized as an important part of a mathematics department’s mission and should be undertaken in collaboration with mathematics education faculty. More mathematics faculty need to become deeply involved in PreK–12 mathematics education by participating in preparation and professional development for teachers and becoming involved with local schools or districts. Mathematics departments need to encourage and reward faculty for these efforts. Departments also need to devote commensurate resources to designing and staffing courses

²Lesson study is a process in which teachers jointly plan, observe, analyze, and refine actual classroom lessons. Math teachers’ circles focus primarily on giving teachers an experience to be learners and doers of mathematics. See the web resources for further information and examples.

for prospective and practicing teachers. For some departments, these courses may require new institutional arrangements. Within a department, courses designed for prospective high school teachers can serve the needs of other mathematics majors to work in ways characteristic of mathematics. Courses designed for practicing teachers can combine intensive multi-day meetings with distance learning. One course can be shared by several institutions, or as part of a regional mathematics education consortium.³ (See the web resources for examples.)

At a minimum, oversight of programs for teachers should be the responsibility of a faculty member with expertise in teacher education as well as mathematics. As for any program, continuity is desirable, for both administrators and instructors. For administrators, this may be of special importance when it requires coordination among academic units (e.g., mathematics, statistics, and education) or with school personnel. For instructors, continuity—and support for professional development—afford increased expertise in teaching teachers.

Although most statistics courses for future teachers are taught by mathematicians or statisticians in mathematics departments, on campuses where there is a separate department of statistics, statistics courses for teachers are seldom a department priority.⁴ This needs to change. Statements for mathematics departments also apply to statistics departments who are responsible for the statistical education of future mathematics teachers.

State departments of education and local school districts recognize the need for continuing education and implement policies requiring professional development or graduate education.⁵ Unfortunately, little of either has been content-based. At the same time, few mathematics or statistics departments provide any opportunities at the graduate level designed to meet the professional needs of PreK–12 mathematics teachers. More mathematics and statistics departments need to have faculty members who work with educators, teachers, and school district personnel to design and implement content-based professional development in schools, districts, and states. National and regional efforts are needed to help prepare these faculty members to contribute effectively to professional development activities for teachers.

There are notable exceptions that can serve as models for departments interested in supporting and serving this important part of the mathematics education community. Examples can be found in the web resources for this report.

Recommendation 6. Mathematicians should recognize the need for improving mathematics teaching at all levels. Mathematics education, including the mathematical education of teachers, can be greatly strengthened by the growth of a mathematics education community that includes mathematicians as one of many constituencies committed to working together to improve mathematics instruction at all levels and to raise professional standards in teaching. It is important to encourage partnerships between mathematics faculty and mathematics education faculty, between

³See Recommendation 13 of *National Task Force on Teacher Education in Physics: Report Synopsis*, American Association of Physics Teachers, the American Physical Society, & the American Institute of Physics, 2010.

⁴In the 2005 CBMS survey, special courses for K–8 teachers were offered by 11% of Ph.D.-granting and 33% of M.A.-granting statistics departments. Less than 0.5% of statistics departments surveyed reported that special sections of regular courses were designated for K–8 teachers. See Table SP.3.

⁵For an overview, see *Key State Education Policies on PK–12 Education: 2008*, Council of Chief State School Officers, p. 22.

faculty in two- and four-year institutions, and between mathematics faculty and school mathematics teachers, as well as state, regional, and school-district leaders.

In particular, as part of the mathematics education community, mathematicians should support the professionalism of mathematics teachers by:

- i. endeavoring to ensure that PreK–12 mathematics teachers have sufficient knowledge and skills upon receiving initial certification;
- ii. encouraging all who teach mathematics to strive for continual improvement in their mathematics teaching;
- iii. joining with teachers at different levels to learn with and from each other.

Strategies for raising professional standards in teaching include developing a career ladder that keeps outstanding teachers in the profession, and providing professional development opportunities for teachers to grow from early career teachers, to mid-career teachers, to master teachers. Although mathematics teachers themselves need to provide leadership, this effort will be enhanced by the development of a comprehensive professional community involving all who teach mathematics or statistics. Society will be better served by focusing efforts on institutional arrangements and professional practices that foster expertise, such as on raising standards at the time of entry into school teaching and providing professional development based on content, rather than relying on punitive approaches focused on weeding out ineffective practicing teachers.

There are many initiatives, communities, and professional organizations focused on aspects of building professionalism in mathematics teaching. More explicit efforts are needed to bridge current communities in ways that build upon mutual respect and the recognition that these initiatives provide opportunities for professional growth for higher education faculty in mathematics, statistics, and education as well as for the mathematics teachers, coaches, and supervisors in the PreK–12 community. The web resources for this report include examples of such collaborative work. Also needed are more opportunities for observation and discussion of the work of teaching, including professional learning communities, math teachers' circles, conferences, and publications, from newsletters to scholarly articles. Mathematicians have an important role to play in all these efforts.

Finally, becoming part of a community that connects all levels of mathematics education will offer mathematicians more opportunities to participate in setting standards for accreditation of teacher preparation programs and for teacher certification via standard and alternative pathways.

