

Chapter 4 footnotes with hyperlinks

The footnotes from Chapter 4 are listed below and hyperlinked (when possible) to the references cited.

Many of the documents cited are freely available. National Research Council reports such as *Adding It Up* can be read on-line. They can be downloaded without charge as can documents from the Conference Board of the Mathematical Sciences and the Council of Chief State School Officers. In some cases, cited portions of documents can be seen via Google Books.

Mathematics education research journal articles are likely to require a subscription. At many academic institutions, these journals will be accessible via institutional subscription. Attempts to access a JSTOR link without such a subscription will get the response “Cannot download the information you requested.”

Note that the MET II web resources at www.cbmsweb.org give URLs for the [CCSS](#), the [Progressions for the CCSS](#), and other relevant information.

1. As noted in Chapter 3, “Although elementary certification in most states is still a K–6 and, in some states, a K–8 certification, state education departments and accreditation associations are urged to require all grade 5–8 teachers of mathematics to satisfy the 24-hour requirement recommended by this report.” Chapters 4 and 5 allow for a period of transition.
2. For example, “It is widely assumed—some would claim common sense—that teachers must know the mathematical content they teach” (*Foundations for Success: Reports of Task Groups of the National Mathematics Advisory Panel*, 2008, [p. 5-6](#)). “Aspiring elementary teachers must begin to acquire a deep conceptual knowledge of the mathematics that they will one day need to teach, moving well beyond mere procedural understanding” (*No Common Denominator*, 2008, National Council on Teacher Quality). “Mathematics courses for future teachers should develop ‘deep understanding’ of mathematics, particularly of the mathematics taught in schools at their chosen grade level” ([Curriculum Foundations Project](#), 2001, Mathematical Association of America). See also *Preparing Teachers: Building Sound Evidence for Sound Policy*, 2010, National Research Council, [p. 123](#).
3. An international comparison of prospective elementary teachers found that 48% of the U.S. teachers did not score above “Anchor point 2.” Teachers with this score often had trouble reasoning about factors, multiples, and percentages. See Tatto & Senk, “[The Mathematics Education of Future Primary and Secondary Teachers: Methods and Findings from the Teacher Education and Development Study in Mathematics](#),” *Journal of Mathematics Teacher Education*, 2011, pp. 129–130. *Preparing Teachers* discusses concern about the adequacy of current teacher preparation in mathematics, especially for elementary teachers. See Chapter 6, especially [p. 124](#).
4. See the National Research Council report [Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity](#) (2009) and the [Counting and Cardinality Progression](#).
5. See the [CCSS](#), pp. 88–89; or the [Operations and Algebraic Thinking Progression](#) for details and

examples of situation and solution equations.

6. For examples of how teachers may construe the base-ten system, see Thanheiser, "[Pre-service Elementary School Teachers' Conceptions of Multidigit Whole Numbers](#)," *Journal for Research in Mathematics Education*, 2009.
7. Beckmann, "[The Community of Math Teachers, from Elementary School to Graduate School](#)," *Notices of the American Mathematical Society*, 2011.
8. For instance, a study of prospective elementary and secondary teachers found that many either did not know that division by 0 was undefined or were unable to explain why it was undefined. On average, the secondary teachers had taken over 9 college-level mathematics courses. Ball, "[Prospective Elementary and Secondary Teachers' Understanding of Division](#)," *Journal for Research in Mathematics Education*, 1990.
9. Lesson study is a process in which teachers jointly plan, observe, analyze, and refine actual classroom lessons.
10. See this chapter's section on mathematics specialists for more discussion about their roles in professional development for teachers.
11. See, e.g., discussion of the use and organization of the blackboard in Lewis, *Lesson Study*, Research for Better Schools, 2002, pp. 97–98.
12. "Productive disposition" is discussed in the National Research Council report *Adding It Up*, pp. 116–117, [131](#)–133.
13. See discussion of support in Masingila et al., "[Who Teaches Mathematics Content Courses for Prospective Elementary Teachers in the United States? Results of a National Survey](#)," *Journal of Mathematics Teacher Education*, 2012.
14. See, e.g., Schoenfeld, "[Working with Schools: The Story of a Mathematics Education Collaboration](#)," *American Mathematical Monthly*, 2009, p. 202.
15. See Fennell, "[We Need Elementary Mathematics Specialists Now, More Than Ever: A Historical Perspective and Call to Action](#)," *National Council of Supervisors of Mathematics Journal*, 2011.

16. In general, a math specialist's roles and responsibilities are not analogous to those of a reading specialist.
17. Examples include the [Vermont Mathematics Initiative](#) (a Math Science Partnership), see [Teaching Teachers Mathematics](#), Mathematical Sciences Research Institute, 2009, pp. 36–38. A 3-year randomized study in Virginia found that specialists' coaching of teachers had a significant positive effect on student achievement in grades 3–5. The specialists studied completed a [mathematics program](#) designed by the [Virginia Mathematics and Science Coalition](#) (also a Math Science Partnership) and the findings should not be generalized to specialists with less expertise. See Campbell & Malkus, "[The Impact of Elementary Mathematics Coaches on Student Achievement](#)," *Elementary School Journal*, 2011.
18. [Standards for Elementary Mathematics Specialists: A Reference for Teacher Credentialing and Degree Programs](#), Association of Mathematics Teacher Educators, 2009.
19. These examples come from Lee & Ginsburg, "[Early Childhood Teachers' Misconceptions about Mathematics Education for Young Children in the United States](#)," *Australasian Journal of Early Childhood*, 2009. This article summarizes research in this area and discusses possible sources of such beliefs.
20. See *Mathematics Learning in Early Childhood*, National Research Council, 2009, pp. [341](#)–343.