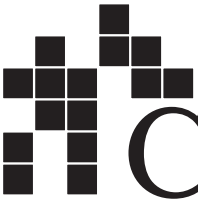
The Mathematical Education of Teachers II



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Writing Team

Sybilla Beckmann, University of Georgia (lead writer, elementary grades chapter)

Daniel Chazan, University of Maryland

Al Cuoco, Education Development Center

Francis (Skip) Fennell, McDaniel College

Bradford Findell, The Ohio State University

Cathy Kessel, Mathematics Education Consultant (editor)

Karen King, National Council of Teachers of Mathematics

W. James Lewis, University of Nebraska–Lincoln (chair)

William McCallum, University of Arizona (lead writer, high school chapter)

Ira Papick, University of Nebraska–Lincoln

Barbara Reys, University of Missouri

Ronald Rosier, Conference Board of the Mathematical Sciences

Richard Scheaffer, University of Florida

Denise A. Spangler, University of Georgia (lead writer, middle grades chapter)

Alan Tucker, State University of New York at Stony Brook (lead writer, Chapters 1–3)

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Preface

This report is a resource for those who teach mathematics—and statistics[1]—to PreK–12 mathematics teachers, both future teachers and those who already teach in our nation's schools. The report makes recommendations for the mathematics that teachers should know and how they should come to know that mathematics. It urges greater involvement of mathematicians and statisticians in teacher education so that the nation's mathematics teachers have the knowledge, skills, and dispositions needed to provide students with a mathematics education that ensures high school graduates are college- and career-ready as envisioned by the Common Core State Standards.

Mathematics teacher education is a complex, interdisciplinary enterprise requiring knowledge of teaching and learning as well as knowledge of mathematics. This argues strongly for a partnership between mathematics educators and those who teach mathematics. Thus, this report will also be an important resource for mathematics educators.

The *Mathematical Education of Teachers* (referred to as MET I in this report) was published in 2001. Since that time much has changed. In particular, the attention given by the mathematics profession to the mathematical education of teachers has increased as more mathematicians and statisticians have taken increasingly active roles in teacher preparation and content-based professional development for current teachers. The Math Science Partnerships (supported by the National Science Foundation and the United States Department of Education) and the NSF's Robert Noyce Teacher Scholarship Program have connected institutions of higher education with K–12 school systems, fostering new partnerships and extending existing collaborations. These and other changes in institutional support and emphasis have helped to increase the engagement of collegiate mathematicians and statisticians in teacher education.

Their engagement has proved to have a wide variety of benefits. For the mathematicians and statisticians, preparation and professional development for teachers can be genuinely interesting intellectual experiences, affording the opportunity to "think deeply about simple things," and to make connections between the undergraduate courses that they teach and K–12 mathematics.

Attending to the needs of prospective teachers by focusing on reasoning and proof across the spectrum of undergraduate mathematics courses that they take, helps them to make sense of mathematics—and makes it easier to understand, easier

[1]In K–12 schools, statistics is part of the mathematics curriculum. At the collegiate level, statistics is recognized as part of the mathematical sciences, but a separate discipline and most research universities have a separate department of statistics. Strengthening PreK–12 mathematics education requires the active involvement of both mathematicians and statisticians.

to teach, and intellectually satisfying for all course-takers. Thus, attending to the needs of future teachers in this way benefits all undergraduates.

For practicing K–12 teachers, content-based professional development offered by Math Science Partnerships has changed their attitudes about mathematics, and increased their mathematical interest and abilities. Moreover, it has increased the achievement of their students.

Determining the most important mathematics that teachers should know requires a clear vision of the mathematics that they will be expected to teach. The Common Core State Standards represent such a vision. Because most states have adopted the Common Core, the recommendations of this report focus on enabling teachers to teach that mathematics.

This report (MET II) draws on the experience and knowledge of the past decade to:

- Update MET I's recommendations for the mathematical preparation of teachers at all grade levels: elementary, middle, and high school.
- Address the professional development of teachers of mathematics.
- Discuss the mathematical needs of teachers with special responsibilities such as elementary mathematics specialists and special education teachers.

At the same time, MET II reiterates and elaborates themes of the first MET report:

- There is intellectual substance in school mathematics.
- Proficiency with school mathematics is necessary but not sufficient mathematical knowledge for a teacher.
- The mathematical knowledge needed for teaching differs from that of other professions.
- Mathematical knowledge for teaching can and should grow throughout a teacher's career.

Chapter 1 describes these themes in more detail, outlining the mathematical issues that underlie the recommendations in this report, including the structure and content of the Common Core. Chapter 2 summarizes empirical findings that underlie these recommendations and connects them with the current educational context. Chapter 3 gives recommendations for strengthening the mathematical education of teachers in the United States, with respect to the mathematics that teachers should learn and the roles of mathematicians and statisticians in their learning. This chapter will be of special interest to department chairs, policymakers, and others in leadership positions.

Chapters 4, 5, and 6 give recommendations for the mathematical preparation and professional development of elementary, middle grades, and high school teachers. These will be the chapters of greatest importance for those engaged in teacher preparation or professional development.

Appendix A gives a short annotated list with two types of entries: recent reports whose conclusions inform the recommendations in this document, and sources of information about accreditation and licensure.

The Common Core State Standards have two categories: those concerning mathematical content and those concerning mathematical practice. Appendix B gives an overview of the content standards. The Standards for Mathematical Practice are given in Appendix C.

Web resources. Web resources associated with this report are located on the web site of the Conference Board of the Mathematical Sciences, `www.cbmsweb.org`. These are intended as an initial collection of relevant information rather than as a continuously updated reference.

Audience. This report should be useful to the entire community of professionals who educate teachers of mathematics, from those who teach undergraduates seeking initial certification to those who work with veteran teachers pursuing opportunities for professional development. Its audience includes professional development providers housed outside of academic institutions as well as collegiate faculty from disciplines outside the mathematical sciences who have become actively engaged in the mathematical education of teachers.

Its primary audiences, however, are faculty who teach in mathematics or statistics departments and their colleagues in colleges of education who have primary responsibility for the mathematical education of teachers. In addition, this report will be useful to policy-makers at all levels who look to the mathematics and mathematics education community for professional guidance with respect to the mathematical education of teachers. Thus, the three main audiences are:

> *Mathematicians and statisticians.* Faculty members of mathematics and statistics departments at two- and four-year collegiate institutions teach the mathematics and statistics courses taken by prospective and practicing teachers. Their departmental colleagues set policies regarding mathematics teacher preparation. At the risk of oversimplification, this report will at times refer to this audience as "mathematicians" or "mathematics faculty."
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> *Mathematics educators.* Mathematics education faculty members, whether within colleges of education, mathematics departments, or other academic units, are also an important audience for this report. Typically, they are responsible for the pedagogical education of mathematics teachers (e.g., teaching methods courses), organizing field experiences for prospective teachers, and for providing overall leadership for the institution's mathematics teacher preparation program. Outside of academe, a variety of people are engaged in professional development for teachers of mathematics, including state, regional, and school-district mathematics specialists. The term "mathematics educators" will include this audience.

Policy-makers. The report is also intended to inform educational administrators and policy-makers at the national, state, school-district, and collegiate levels as they work to provide PreK–12 students with a strong mathematical preparation for the increasingly quantitative workplace. Teachers' knowledge of mathematics is central to this effort, thus, institutions of higher education have a key role to play in teachers' professional development as well as their preparation.

Teachers' learning of mathematics is supported—or hindered—by institutional policies. These include national accreditation requirements, state certification requirements, and the ways in which they are reflected in teacher preparation programs. State and district supervisors make choices in provision and funding of professional development. At the school level, scheduling and policy affect the types of learning experiences available to teachers. Thus, policy-makers play important roles in the mathematical education of teachers.

Terminology. To avoid confusion, the report uses the following terminology:

Student refers to a child or adolescent in a PreK–12 classroom.

Teacher refers to an instructor in a PreK–12 classroom but may also refer to a prospective PreK–12 teacher in a college mathematics course ("prospective teacher" or "pre-service teacher" is also used in the latter case).

Instructor refers to an instructor of prospective or practicing teachers. Because this report concerns the roles of mathematicians in teacher education, "instructor" will usually refer to a mathematician.

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The content and exposition of this report has benefited from extensive and thoughtful criticism of an earlier draft from teachers, mathematicians, and mathematics educators.

That earlier draft drew on comments and suggestions made by participants at the 2010 CBMS Forum on Content-Based Professional Development and the 2011 CBMS Forum on Teaching Teachers in the Era of the Common Core. These were made possible by support from the Brookhill Foundation and the National Science Foundation.