Chapter 1

Changing Expectations, New Realizations

This report calls for a rethinking of the mathematical education of prospective teachers within mathematical sciences departments at U. S. two- and four-year colleges and universities. It offers principles to assist departments in this process, along with specific suggestions for mathematics courses for prospective teachers. Additionally, this report seeks to convince faculty that there is more intellectual content in school mathematics instruction than most realize, content that teachers need to understand well.

Opinion polls show that education currently is the number one concern of American adults. A 1998 Harris Poll revealed that roughly nine out of ten Americans believe that the best way to raise student achievement is to ensure a qualified teacher in every classroom. States are mandating higher standards for teacher certification, as well as for student achievement, and they are monitoring teacher preparation programs more closely. School mathematics instruction and the mathematical preparation of teachers are in the spotlight, because, after reading and writing, mathematics is widely viewed as the most important component of K–12 education to promote future success in college and subsequent careers. All mathematicians[1] should be concerned about teacher education, and all have a role to play in setting policies, if not offering instruction, in the mathematical education of teachers.

It goes without saying that mathematicians want prospective teachers to have a solid understanding of the mathematics that they will teach. The daunting challenge is how to achieve this goal, given the diverse mathematical preparation of college students who will become teachers and the changing views about what mathematical knowledge is needed to be an effective teacher. Mathematicians who teach introductory mathematics courses commonly encounter too many students who are afraid of mathematics, lack needed study skills, or have deficiencies in their mathematical background. On top of these problems, mathematicians who teach courses for prospective teachers face the additional challenge of making appropriate connections to mathematical instruction in school classrooms.

A number of mathematicians and mathematics education researchers have recognized the special nature of the mathematical knowledge needed for K–12 teaching and its implications for the mathematical preparation of teachers. In particular,

[1]For simplicity of language, this report will use the terms "mathematics" and "mathematicians" to refer to the mathematical sciences, including statistics, and to mathematical scientists, respectively. Likewise, "mathematicians" refers to faculty at two-year colleges, four-year colleges, and universities.

the interviews with Chinese elementary teachers in Liping Ma's 1999 book *Knowing and Teaching Elementary Mathematics* awakened many mathematicians to this issue and its mathematical substance. For example, mathematical knowledge for teaching should prepare an elementary teacher to assess the validity of the following subtraction procedure proposed by a student:

$$\begin{array}{rcll} 43-27 & = & (40-20) \text{ (tens place subtraction)} & + \ (3-7) \text{ (ones place subtraction)} \\ & = & 20 & + \ -4 \\ & = & 16 & \end{array}$$

and to appreciate the complications of trying to extend it to larger numbers.

Some aspects of mathematical knowledge for teaching, such as those illustrated in the example above, may seem to mathematicians to fall into the domain of methods courses in education. However, education faculty generally see these issues to be more appropriately addressed in mathematics courses, and so such issues often remain unaddressed in teacher preparation. This state of affairs is one of many reasons why efforts to improve the mathematical education of teachers require a partnership between faculty in mathematics and mathematics education.

Changing expectations for mathematics knowledge. Arithmetic skills, and occasionally a little algebra, were once the mathematics required for almost all jobs outside of engineering and the physical sciences. In recent years, computers and an associated explosion in the use of quantitative methods in business and science have dramatically increased the mathematical skills needed in many jobs. Facility at creating spreadsheets is becoming required in many entry-level positions for high school graduates. Assembly line workers may be expected to learn elements of statistical quality control. The level of mathematical sophistication common in financial analyses today would have been unthinkable a generation ago.

Changing expectations for school mathematics instruction. Public education in the United States has historically had a utilitarian focus, which in mathematics, emphasized arithmetic skills and problems from commerce, such as compound interest. Until recently, only high school students in college preparatory tracks studied algebra, and then often just for one year. In the two decades after World War II, there were efforts to increase the sophistication of mathematics curricula in colleges and schools. These efforts succeeded in moving calculus into the first year of college and modernizing the curriculum of mathematics majors. The school mathematics reform efforts of the 1960s and 1970s had a number of long-lasting influences, such as broadening elementary school mathematics beyond arithmetic to include some geometry and elements of algebra, and refocusing high school mathematics by downplaying analytic geometry and trigonometry and giving more attention to functions and providing an introduction to calculus. However, this period is most remembered for the New Math movement's theoretical approach. This approach was widely rejected, leaving school mathematics reform efforts on the defensive for many years to come.

There have been complaints about the poor mathematical skills of students throughout the history of U. S. education. An unacceptably low number of students today are making their way up the mathematical ladder to the higher levels of mathematical knowledge that their college majors and future employment require. There are growing concerns about what students are being taught, as well as how well they are learning it. Since the 1980s, international comparison studies, such as

the Third International Mathematics and Science Study, have indicated that many developed countries, particularly some in east Asia, provide school mathematics instruction richer than that of the United States (see, e.g., Stevenson and Stigler's book *The Learning Gap* and McKnight et al.'s *The Under-achieving Curriculum*).

In 1989, the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards* initiated an overdue period of intense interest in strengthening school mathematics instruction, to make it both more demanding and more effective for all students. Given the immense problems in both primary and secondary schools which affect both the development of procedural and reasoning skills, it was probably inevitable that there would not be a consensus on the precise nature of some shortcomings in current mathematics instruction, nor on proposed solutions. In 2000, the NCTM *Principles and Standards for School Mathematics*, an update of the 1989 *Standards*, addressed the issues of interpretation and implementation that had emerged over the preceding decade, including input from a broad spectrum of mathematical science and mathematics education organizations.

New realizations about the mathematical education of teachers. Throughout U. S. educational history, teachers have generally provided the style and level of instruction that society expected of them. Until 1900, teachers of mathematics were largely seen as drill masters, training students to accurately perform numerical computations. Beyond the eight primary grades, most teachers had at best a year or two of preparation at a special high school, called a normal school. The introduction of universal high school around 1900 gave rise to secondary level subject specialists, who majored in their subject in teachers' colleges. Teachers for earlier grades also were eventually required to go to college, but their education focused on the psychological and social development of children. It was generally assumed, and is still assumed by some today, that prospective elementary school teachers, and perhaps middle school teachers, learn all the mathematics they need to teach mathematics well during their own schooling.

Recently, this assumption has been seriously questioned. There is evidence of a vicious cycle in which too many prospective teachers enter college with insufficient understanding of school mathematics, have little college instruction focused on the mathematics they will teach, and then enter their classrooms inadequately prepared to teach mathematics to the following generations of students. Studies of teachers' mathematical knowledge, for example Ball's 1991 "Research on teaching mathematics," have drawn attention to substantial mathematical issues that arise in day-to-day school instruction, but are not well understood by prospective U. S. teachers, when they graduate from college. One example is the place value structure of our number system, which implicitly expresses numbers as polynomials in powers of 10 and permits single-digit arithmetic to be easily extended to multi-digit arithmetic (in contrast with Roman numeral computations). Elementary teachers need a deep understanding of place value to help their students use it as a foundation for the successful learning of integer arithmetic, and later decimal arithmetic and symbolic calculations in algebra. Likewise, middle grades teachers need a deep understanding of proportions to help their students master fractions, and high school teachers need a deep understanding of functions to help prepare their students for the future study of calculus.

International studies have also highlighted the importance of continuing study as an integral part of a teacher's weekly duties. Thus, college mathematics courses should be designed to prepare prospective teachers for the lifelong learning of mathematics, rather than to teach them all they will need to know in order to teach mathematics well.

In sum, these new expectations and realizations make a strong case for a thorough rethinking of mathematics courses for prospective teachers of all grade levels.