

Chapter 2

General Recommendations

This chapter presents the core recommendations to mathematics departments and the general mathematics community about the mathematical education of teachers. These recommendations fall into three categories: curriculum and instruction for prospective teachers in mathematics departments, cooperation between mathematics departments and other parties involved with the education of teachers, and mathematicians' involvement in national policy that supports high quality mathematics teaching.

Mathematics Curriculum and Instruction for Prospective Teachers

Recommendation 1. Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach. The mathematical knowledge needed by teachers at all levels is substantial, yet quite different from that required by students pursuing other mathematics-related professions. Prospective teachers need to understand the fundamental principles that underlie school mathematics, so that they can teach it to diverse groups of students as a coherent, reasoned activity and communicate an appreciation of the elegance and power of the subject. With such knowledge, they can foster an enthusiasm for mathematics and a deeper understanding among their students. College courses developing this knowledge should make connections between the mathematics being studied and mathematics prospective teachers will teach. Finally, prospective teachers need to develop a thorough mastery of the mathematics in several grades beyond that which they expect to teach, as well as of the mathematics in earlier grades.

Courses about the fundamental ideas of school mathematics should be taught by mathematicians who have a serious interest in teacher education. This instruction should be coordinated with faculty in mathematics education. It is vital that faculty in the mathematics department and the school of education agree on substantial expectations for student learning and achievement in these mathematics courses. Further, careful thought needs to be given to appropriate prerequisite knowledge for these courses. Special courses for teachers, rather than traditional courses, such as intermediate algebra and college algebra, are desirable for those lacking prerequisites.

Recommendation 2. Although the quality of mathematical preparation is more important than the quantity, the following amount of mathematics coursework for prospective teachers is recommended.

- (i) *Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.*
- (ii) *Prospective middle grades teachers of mathematics should be required to take at least 21 semester-hours of mathematics, that includes at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.*
- (iii) *Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics, that includes a 6-hour capstone course connecting their college mathematics courses with high school mathematics.*

Chapters 3–5 give recommendations for mathematics courses for prospective elementary, middle, and high school teachers. Chapters 7–9 (Part 2 of this report) develop these recommendations in greater detail.

Recommendation 3. Courses on fundamental ideas of school mathematics should focus on a thorough development of basic mathematical ideas. All courses designed for prospective teachers should develop careful reasoning and mathematical “common sense” in analyzing conceptual relationships and in solving problems. Attention to the broad and flexible applicability of basic ideas and modes of reasoning is preferable to superficial coverage of many topics. Prospective teachers should learn mathematics in a coherent fashion that emphasizes the interconnections among theory, procedures, and applications. They should learn how basic mathematical ideas combine to form the framework on which specific mathematics lessons are built. For example, the ideas of number, geometry, and function, along with algebraic and graphical representation of information, form the basis of most high school algebra and trigonometry.

Recommendation 4. Along with building mathematical knowledge, mathematics courses for prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching. Mathematics is not only about numbers and shapes, but also about patterns of all types. In searching for patterns, mathematical thinkers look for attributes like linearity, periodicity, continuity, randomness, and symmetry. They take actions like representing, experimenting, modeling, classifying, visualizing, computing, and proving. Teachers need to learn to ask good mathematical questions, as well as find solutions, and to look at problems from multiple points of view. Most of all, prospective teachers need to learn how to learn mathematics.

Results of international studies, as described, for example, in Stigler and Hiebert’s 1999 book *The Teaching Gap*, indicate that U. S. school mathematics instruction places a comparatively low priority on engaging students to develop an understanding of mathematics. To foster more of this activity in schools, prospective teachers need to experience such instruction in their college mathematics classes and to learn that there are multiple ways to engage students in mathematics.

Recommendation 5. Teacher education must be recognized as an important part of mathematics departments’ mission at institutions that educate teachers. More

mathematicians should consider becoming deeply involved in K–12 mathematics education. Mathematics departments should devote commensurate resources to designing and offering courses for teachers. They should also value and properly reward the faculty members heavily involved in teacher education. Whether or not a mathematics department contains a group of mathematics education specialists, it is important for the entire mathematics faculty to actively support teacher education efforts. In return, mathematics departments should receive the resources needed to follow through on their commitment to high quality teacher education. As argued in the 1999 American Mathematical Society report *Towards Excellence*, a strong commitment to issues of societal concern, such as teacher education, can help garner administrative support for other departmental priorities.

Senior mathematics faculty, who have become involved in K–12 mathematics education, should see themselves as possible models for fellow faculty, and their efforts should be publicized. All mathematics faculty should value, encourage, and support interest among students in pursuing a career in teaching.

Cooperation Among Parties Involved in Teacher Education

Recommendation 6. The mathematical education of teachers should be seen as a partnership between mathematics faculty and mathematics education faculty. Most good school mathematics instruction involves a combination of mathematical knowledge and pedagogy, such as choosing appropriate examples and teaching strategies for various topics. Mathematics educators can provide valuable insights and information about what takes place in school classrooms, including common mathematical misunderstandings of practicing teachers and how to build on these to promote real understanding. They have access to information on state curriculum guidelines and research studies about teachers' mathematical knowledge. In return, mathematics faculty can help mathematics education faculty by keeping them informed of mathematical developments which have an impact on school mathematics.

Ph.D.-granting mathematics departments are encouraged to work with mathematics education faculty to develop new Ph.D. programs in mathematics education as a response to the many unfilled faculty openings for mathematics education specialists. These two groups are also encouraged to develop “minors” in mathematics education for mathematics Ph.D.'s, similar to the statistics and computer science concentrations that some mathematics Ph.D. programs offer.

The reality today is that there is considerable distrust between mathematics faculty and mathematics education faculty both within institutions and through public exchange. Conscious efforts, locally and nationally, are needed to foster cooperation, along with mutual understanding and respect between these two groups. Mathematicians and mathematics educators, when working cooperatively, can be more effective in influencing the state and national organizations responsible for curriculum standards and certification of teacher education programs.

Recommendation 7. There needs to be greater cooperation between two-year and four-year colleges in the mathematical education of teachers. Two-year colleges are essential partners in the mathematical education of teachers. A large number of future teachers begin their post-secondary study in two-year colleges. In particular, many elementary teachers take a significant portion of their college mathematics courses at two-year colleges. Many of the new courses on school mathematics,

mentioned in Recommendation 2 and discussed in subsequent chapters, should be offered in two-year colleges.

Moreover, there are special difficulties that two-year colleges face in helping to prepare teachers. Students of two-year colleges transfer to a variety of different four-year institutions with differing course requirements for prospective teachers. Good articulation agreements are needed among two-year colleges, university mathematics departments, and education departments about the mathematical coursework for prospective teachers. The 1999 National Science Foundation report *Investing in Tomorrow's Teachers* contains a comprehensive set of recommendations about the role of two-year colleges in the mathematical education of teachers.

Recommendation 8. There needs to be more collaboration between mathematics faculty and school mathematics teachers. Observing teachers in action and learning about their experiences can give mathematicians helpful perspectives for their instruction of prospective teachers. Involving school teachers in courses about school mathematics is a tangible way to connect the courses with real practice. In turn, mathematicians should have an important role in professional development activities for teachers.

Policies to Support High Quality School Mathematics Teaching

Recommendation 9. Efforts to improve standards for school mathematics instruction, as well as for teacher preparation accreditation and teacher certification, will be strengthened by the full-fledged participation of the academic mathematics community. While it is generally true that few academic mathematicians gave much attention to standards for school mathematics and teacher preparation in the 1970s and 1980s, they have historically played a major role in these arenas, and in recent years, a number have become involved again. Published standards are likely to carry more weight and generate less controversy when mathematicians, along with mathematics education researchers, curriculum developers, teachers, and other interested parties, play a significant role in their drafting.

In terms of the agenda of this report, it is hard for mathematicians to develop good programs in the mathematical education of teachers, if they are not involved in establishing standards for school mathematics instruction, accreditation of teacher preparation programs, and teacher certification.

Recommendation 10. Teachers need the opportunity to develop their understanding of mathematics and its teaching throughout their careers, through both self-directed and collegial study, and through formal coursework. In some countries where student achievement is high, teachers, alone and in groups, spend time refining their lessons and studying the underlying mathematics. They observe each other's classes. Beginning teachers have extensive mentoring. The teachers' manuals accompanying their textbooks have extensive background material about the mathematics being taught and how it fits into the overall curriculum. More professional development opportunities of this kind are essential for U. S. teachers.

Through continuing education courses, outreach to schools, and other efforts, mathematics departments should support a culture in the teaching profession that promotes professional growth. Mathematicians should speak out in support of

changes in the schools to make professional development an integral part of a teacher's job.

A more complete discussion of the role of mathematics faculty in professional development concerns is left to a future study.

Recommendation 11. Mathematics in middle grades (grades 5–8) should be taught by mathematics specialists. This recommendation mirrors similar recommendations by a number of other groups seeking to improve U. S. school mathematics instruction. Middle grades mathematics teachers must know the high school mathematics curriculum well and understand the foundation that is being laid for it in their instruction. As concepts like fractions and decimals enter the curriculum, teaching mathematics well requires subject matter expertise that non-specialists cannot be expected to master. Having mathematical specialists, beginning in middle grades, both reduces the educational burden for those teaching mathematics in these grades and provides opportunities for prospective teachers of these grades who like mathematics to specialize in it.