

Preface

This report is designed to be a resource for mathematics faculty and other parties involved in the education of mathematics teachers. It is a distillation of current thinking on curriculum and policy issues affecting the mathematical education of teachers, with the goal of stimulating efforts on individual campuses to improve programs for prospective teachers. It is also intended to marshal the backing of the mathematical sciences community for important national initiatives, such as the use of mathematics specialists to teach mathematics starting in middle grades and expanded time for professional development in the schools.

Now is a time of great interest in K–12 mathematics education. Student performance, curriculum, and teacher education are the subjects of much scrutiny and debate. Studies of the mathematical knowledge of prospective and practicing U. S. teachers suggest ways to improve their mathematical educations.

Two general themes of this report are: (i) the intellectual substance in school mathematics; and (ii) the special nature of the mathematical knowledge needed for teaching. It is often assumed that, because the topics covered in school mathematics are so basic, they must also be easy to teach. We owe to mathematics education research of the past decade, or so, the realization that substantial mathematical understanding is needed even to teach whole number arithmetic well. Several mathematics education researchers, in particular Deborah Ball and Liping Ma, have been able to communicate these findings in ways that engaged research mathematicians. Middle grades curricula are even more demanding; for example, the structure of the rational numbers and the idea of proportionality require even more knowledge of teachers. High school mathematics is often considered more substantive than the mathematics of earlier grades, but the challenges of developing a knowledge of it for teaching are often unacknowledged.

The mathematical knowledge needed for teaching is quite different from that required by college students pursuing other mathematics-related professions. Prospective teachers need a solid understanding of mathematics so that they can teach it as a coherent, reasoned activity and communicate its elegance and power. Mathematicians are particularly qualified to teach mathematics in the connected, sense-making way that teachers need. For maximum effectiveness, the design of this instruction requires collaboration between mathematicians and mathematics educators and close connections with classroom practice.

This report is not aligned with a particular school mathematics curriculum, although it is consistent with the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* as well as other recent national

reports on school mathematics. This report focuses on preservice education. Although there is a growing awareness that teachers need more professional development opportunities, this project did not have the time or resources to discuss this important issue in detail.

This report is addressed to a number of different audiences. The following paragraphs offer guidance to readers in some of these audiences.

Mathematics faculty. The primary audience for this report are the members of the mathematics faculties of two-year colleges, four-year colleges, and universities. In this report, the terms “mathematicians” and “mathematics faculty” refer to all mathematical scientists, including statisticians. Teacher education is an important component of the educational mission of most mathematics departments. Many faculty members of these departments, not just the teacher education specialists, ought to have an appreciation of the curriculum and policy issues that affect prospective and practicing teachers. Part 1 of this report, Chapters 1–6, attempts to give mathematics faculty a useful summary of these issues. In departments where some mathematics majors are prospective high school mathematics teachers, mathematicians who teach courses in the major are also encouraged to read Chapter 9 for information about the needs of prospective teachers in their courses.

Mathematics education faculty and mathematics faculty deeply involved in teacher education. Part 2 of this report, Chapters 7–9, is aimed foremost at these faculty. These chapters go into considerable detail about major themes in these courses, including strategies for developing a sound mathematical understanding and an awareness of common student misconceptions. On the other hand, these chapters do not spell out complete topic-by-topic syllabi for mathematics courses for teachers. The creation of revised and new courses for teachers should be the product of extensive local and national discussion and experimentation, built on the ideas in these chapters. Considerable variation from institution to institution is likely.

Mathematics chairs and deans. The general recommendations in Chapter 2 are the most important part of the report for these readers. Mathematics chairs also need to be familiar with the themes of Chapters 3–5, which summarize recommendations about the mathematics courses for prospective teachers at different grade levels.

Mathematics supervisors in schools, district offices, and state education departments. Although the primary audience for this report is mathematics faculty, it is important that the mathematical education of teachers be recognized both inside and outside of colleges and universities as a collaboration involving all interested parties. Parts 1 and 2 of this report are meant to build a foundation for such collaboration.

Education policy bodies at the state and national level. Policy issues are summarized in Chapter 2. This chapter is meant to serve as a two-way street between mathematicians and state and national education policy bodies. It presents the policy concerns of mathematicians, which need support from

policy bodies, and recommendations from these bodies, which mathematicians are encouraged to support.

Organizations of mathematicians. Professional organizations in the mathematical sciences have a critical role to play in the mathematical education of teachers, by fostering discussion and encouraging greater involvement among their members. They can organize workshops on this topic at professional meetings and maintain Web sites with information on teacher education and related issues.

Organizations that fund efforts to improve teacher education. This report asks mathematicians to rethink courses on fundamental ideas of school mathematics, and in particular, that they give more attention to the special nature of the mathematical knowledge needed for teaching. It is imperative that federal agencies and private foundations fund a wide array of efforts to develop successful models of such courses and support independent assessments of such models. Support for workshops to disseminate their results is also necessary.

Accreditation and certification organizations. Agencies that certify teachers and accredit teacher education programs need to work in careful coordination with mathematics and mathematics education faculties, as the rethinking suggested here evolves. This report does not offer detailed standards for teacher knowledge of mathematics that can be readily translated into certification requirements. However, it does provide considerable guidance about the kind of mathematical understanding that teachers need and the connection of that understanding with day-to-day classroom instruction.

The writers of this report hope that it will be useful to other audiences—both current and future teachers, education faculty in general, and school supervisors of mathematics.

This document is available in print and electronic formats. Part 1 is published by the Mathematical Association of America and may be obtained free of charge from the Conference Board of the Mathematical Sciences. The American Mathematical Society has published Parts 1 and 2 as a single volume. Parts 1 and 2 may also be downloaded from the Conference Board of the Mathematical Sciences Web site, <http://www.maa.org/cbms>.

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