Getting Connected with K-12 Teachers

William Fisher & Margaret Owens
California State University, Chico

Abstract: California State University, Chico has a long tradition of providing high quality teacher training programs dating back to its beginning as a normal school. The Department of Mathematics and Statistics values this tradition and is committed to its service to future teachers. With funding from the ExxonMobil Foundation we have been able to establish a Teacher-in-Residence program that provides an exciting link between the university and K-6 education. Building upon that model, we have extended our collaborations to secondary education. We will present an overview of some of our on-going work with K-12 teachers, including the ramifications for the mathematical preparation of future elementary and high school teachers, and for in-service teachers as well. Our focus will be on ways in which these connections with the K-12 mathematical community can be established and ways in which such connections can improve teacher education programs.

California State University, Chico is one of twenty-three campuses in the California State University system. Located in a rural area approximately one hundred miles north of Sacramento, CSU, Chico is a residential campus of nearly 16,000 students. Founded in 1887 as a state normal school, the campus takes pride in its long tradition of professional development for both preservice and inservice teachers. Teacher training continues to be an important component of the campus mission. For example, at the present time over 1400 students are liberal studies majors, the most popular major on campus and the major most students elect to earn a multiple subjects credential, the credential needed to teach in the elementary schools.

The Department of Mathematics and Statistics is the largest on campus in terms of number of students served. We have a healthy number of mathematics and statistics majors, but we are primarily a service department. Enrollment in our courses seldom exceeds thirty-five students. The department consists of twenty-one tenured or tenure-track faculty, about one-third of whom are actively engaged in work in mathematics education. We rely heavily on part-time faculty members, currently employing forty-five adjunct faculty who teach a variety of courses ranging from remedial mathematics to mathematics for future elementary teachers to calculus.

Elementary School Connections

<u>Future Elementary Teachers</u>

The model for teacher credentialing in California requires students to earn a bachelor's degree in a subject area and then enter into a "fifth year" practicum, which includes the student teaching experience. Students who choose to become elementary teachers usually become Liberal Studies majors, a cross-curricular major that the university has created for these students. Nevertheless, students wanting to enter student teaching in elementary education can also major in disciplines such as child development, psychology, etc. as long as they still meet mandated subject area requirements. These requirements currently include six semester units of lower-division mathematics. The mathematics department offers a standard year-long course that parallels the content of traditional mathematics for elementary teachers texts such as those by Long &

DeTemple, Musser & Burger, and Bassarear, to name a few. There is a movement to add another three units at the upper-division level.

Many future elementary teachers in our program already complete more than the required six units of mathematics. Liberal studies majors are required to take at least one upper division class from the College of Natural Science. Possibilities include two mathematics courses we have created specifically for these students or various science classes that all include a lab. For many students, aversion to mathematics is only exceeded by their fear of science and hence most liberal studies students take three semesters of mathematics. Moreover, all liberal studies majors must choose an area of concentration. While child development is the most popular area of concentration, mathematics is second most popular, with nearly 18% of liberal studies majors choosing a mathematics concentration. The concentration in mathematics requires a total of eighteen units, including courses in trigonometry and statistics.

California does not have a separate credential program for middle school teachers. There are two paths to teaching middle school mathematics: earn a bachelor's degree in mathematics and a single subject credential or earn a bachelor's degree in liberal studies, a multiple subject credential, and add a supplementary authorization in introductory mathematics. A supplementary authorization added to a multiple subjects credential allows the holder to teach mathematics in departmentalized classes in grades 6 through 9. The supplementary authorization requires a total of twenty semester units of mathematics (or ten units of upper-division mathematics). The statewide shortage of teachers in mathematics is a much-publicized problem that leads many preservice and inservice teachers to pursue a supplementary authorization in mathematics.

<u>The Teacher-in-Residence Program</u>

The most significant elementary-level partnership we have is a Teacher-in-Residence program that is partially funded by the ExxonMobil Foundation. The idea of this program is to have an elementary school teacher take a leave of absence from his or her classroom to work half-time in the mathematics department at the university and half-time for the district as a mathematics resource specialist. The program has many goals, but an encompassing goal is to develop better links between the university and the K-6 mathematical community. At the university level, we hope to produce college graduates who are better prepared to teach elementary school mathematics, while we also help university faculty better understand the needs of future teachers. At the elementary school level, we hope the program will help classroom teachers develop a better understanding of mathematics and that this better understanding will produce elementary school students with a deeper knowledge of mathematics.

Teaching in the mathematics department is a major component of the Teacher-in-Residence Program. Our Teacher-in-Residence teaches two sections of mathematics courses for future elementary teachers. However, if this person only taught two sections of mathematics courses, the Teacher-in-Residence would be no different than any of the other multitude of part-time instructors who teach in the department. Consequently, in addition to teaching, the university duties of our Teacher-in-Residence include activities designed to involve our full-time faculty and increase their awareness and understanding of the mathematical needs of elementary teachers.

Team teaching provides a strong connection between the Teacher-in-Residence and one or more of the university faculty. Each semester, one of our full-time faculty team-teaches a section of mathematics for elementary teachers with our Teacher-in-Residence. The team usually meets several times a week to plan and discuss and reflect upon the coursework; other faculty teaching the same course sometimes attend these meetings as well. The Teacher-in-Residence gives a unique and extremely beneficial perspective to these discussions. Collaborations between university faculty and the Teacher-in-Residence are proving so fruitful for our faculty that we provide an opportunity for all new mathematics education faculty to team-teach one course with our Teacher-in-Residence.

While team teaching allows both the university faculty member and the elementary school teacher to grow professionally, it also gives our university students a much richer classroom experience. For example, since the Teacher-in-Residence has immediate access to elementary children, oftentimes a mathematics problem is assigned to both the university students and to an appropriate grade of elementary children. Once the university students have worked on the problem, they are then given elementary children's work on the very same problem! Subsequent class time spent analyzing student work provides the university students an early contact with children's mathematical thinking that has rarely been part of our classes in the past. These child-centered activities validate the necessity of the mathematics coursework and foster a level of excitement that keeps university students engaged in and enthusiastic about mathematics. (See Appendix A for an example of a joint elementary mathematics problem.)

While providing easy access to the work of elementary students is an important contribution, our Teacher-in-Residence has been able to establish an even stronger link between our preservice mathematics education courses and the elementary classroom. Most sections of mathematics for elementary teachers now have an elementary class visit their university class at least once per semester. Each time, we learn more about how to make these occasions most profitable for the university students and not just a fun opportunity to work with children. The Teacher-in-Residence arranges the visit, provides expert help in designing appropriate activities for the visiting class and attends the visit to act as a facilitator between the university and the elementary environments. While scheduling and other details can sometimes be a chore, the value of the experience to our preservice teachers makes the effort worthwhile.

Thanks to interactions with our Teacher-in-Residence, university faculty who teach the mathematics courses for preservice elementary teachers have become much more professionally active in this area. We have initiated a mathematics education seminar series that meets to share ideas, discuss mutual concerns, and become a learning community. Last year we read Liping Ma's *Knowing and Teaching Elementary Mathematics*, much like a book club activity where we covered the book chapter by chapter with appropriate discussion questions. We invited several key elementary school teacher-leaders to become part of this "book club" and all of us gained a profound understanding of elementary mathematics, to use the coined phrase.

In June 2001, Chico was able to send a team to San Diego for the American Association of State Colleges and Universities meeting on Improving the Mathematics Preparation of Elementary Teachers; there was never any question about whether our

Teacher-in-Residence should be part of the team. Our Teacher-in-Residence is a valued member of the department who has insights into the needs of preservice and inservice teachers that are not possible for those of us who work outside of the elementary school community. These insights have improved not just our work in individual courses, but have lead us to rethink our mathematics program for prospective elementary teachers. We have decided that it would be advantageous to restructure the program and include another course offering, one centered on proportional reasoning. Our Teacher-in-Residence program played a key role in stimulating these discussions of curricular issues.

While the contributions of our Teacher-in-Residence certainly seem more significant than what we would expect from half a faculty position, this person also has another "half" position at the Chico Unified School District. This dual role is possible, in part, because the Teacher-in-Residence has a two (or three) day-per-week teaching schedule; for the rest of the week, the Teacher-in-Residence works as a mathematics resource specialist. Rather than working for the entire school district, he or she works at a particular school site. Different sites have been identified as needing an infusion of mathematics reevaluation based on statewide mandated testing. The Teacher-in-Residence has an office at that site and works as a peer coach, does requested teach-ins in various classrooms, conducts "lesson study" professional development workshops with the teachers at that site on a bi-weekly basis, and acts as an on-site resource for all mathematics curricular issues.

As a result of the two different jobs the Teacher-in-Residence occupies, the blended benefits to both the district and the university are numerous and robust. The student-centered benefits are readily apparent. In addition to the visits from elementary school classes mentioned above, we now have an extensive mathematics tutor program at various elementary schools. The program uses preservice teachers who go out into the schools and tutor in mathematics. The university students attend a bi-weekly seminar where they are given very detailed ideas, activities and assessment tasks. They work closely with the classroom teachers to help individual students more fully understand specific mathematical ideas. The seminars are co-taught by the Teacher-in-Residence who also serves as on-site coordinator for the tutors.

Some of the other blended benefits of the program are less easily defined, but there seems to be a much better working relationship between the district and the university because of the Teacher-in-Residence program. More teachers in the district, especially those who bring students to campus, have a better understanding of the university and the mathematics program for preservice teachers. Conversely, mathematics faculty have a much deeper appreciation of the elementary teacher.

It is in fact because of the crossover and blending between the university and the elementary school components and benefits that we are so fully committed to the Teacher-in-Residence program and strongly advocate such a position for all mathematics departments attempting to more effectively provide mathematics instruction for future elementary teachers.

Secondary School Connections

Future Secondary Mathematics Teachers

Approximately 120 of our 150 mathematics and statistics majors plan to teach high school mathematics. Majors in our mathematics education option take all the courses typically required of students majoring in mathematics. In fact, when the rare senior in the program decides that he or she would rather go to graduate school in mathematics than teach high school, we can assure that student that he or she is well prepared for this new direction. Instead of the MET-recommended capstone course for prospective high school teachers, our program offers a sequence of three courses that blend mathematical content and pedagogy and are designed to incorporate the NCTM Professional Teaching Standards. The first course in the sequence is a required junior-level course focusing on problem-solving. In the second course in the sequence, the emphasis shifts from mathematical content to pedagogy; students have the opportunity to present mathematics to other students in the course. While the second course is not required, it satisfies an elective requirement, and most students in the mathematics education option choose to take the course. Team-teaching sections of mathematics content courses for elementary teachers with our Teacher-in-Residence has been so effective that we have also enlisted the services of a secondary teacher to team-teach the second semester of the sequence of courses for preservice secondary mathematics teachers. We offer this course late in the day so that a secondary teacher may co-teach this course after their normal teaching duties at the district level.

The most innovative aspect of our program for future high school mathematics teachers occurs in the third semester of the sequence when our undergraduate students have the opportunity to apply for teaching internships in our department. In this program, our undergraduate mathematics majors teach a reform-based course in remedial mathematics to incoming freshmen who fail to pass an Entry Level Mathematics (ELM) test. Interns teach in teams of two, with each team having complete responsibility for a five-unit course. A senior faculty member visits and critiques each team's class at least twice a week. Interns are required to enroll in a four-unit seminar in which the interns and their supervisor can process what is happening in the interns' classes. After their experience in our internship program, many of our students by-pass the traditional fifth year credential program and obtain internships in the public school system. In these internships, the credential candidate has the opportunity to teach while working on credential requirements.

Secondary Partnerships: Project MATH

Our partnerships at the secondary level have essentially the same goals as the elementary partnerships. At the university level, we hope our undergraduates will be better prepared to teach secondary school mathematics and our university faculty will acquire better understanding of the needs of future teachers. At the secondary level, we hope the program will create a professionalism that extends beyond the program's mentor teachers and we hope to improve the performance of secondary school students. Our all-encompassing goal is to provide better links between the university and secondary schools.

We highlight two partnerships that we have at the secondary level. The first involves our newly developed secondary mathematics teacher preparation program at California State University, Chico. Project MATH (Mathematics And Teaching on the

Horizon) has been created with grants from both NSF and FIPSE. Students who are in Project MATH earn a traditional Bachelor of Science degree in mathematics with a mathematics education option, but they gain many new and unique experiences that better prepare them to become mathematics teachers. In fact, they enroll in a special mathematics education seminar in each semester of their undergraduate career. Project MATH is based on a cohort model; entering freshmen live in a thematic residence house that has double occupancy bedrooms sharing common kitchen and living quarters. Freshmen in the program are usually enrolled in the same section of beginning calculus and have tutoring, conducted by a senior mathematics education major, available in their house three nights a week.

Partnership activities with area secondary teachers are exemplified by the fact that each Project MATH freshman is assigned a middle or secondary mathematics teacher as a mentor teacher for their four years of undergraduate study. Mentor teachers also attend and help facilitate the mathematics education bi-weekly seminars. The seminars focus on topics that are not normally seen by undergraduates before a typical senior capstone seminar, if at all. Time is devoted to carefully discussing, analyzing and modeling the NCTM's *Principles and Standards for School Mathematics*. We use the *Professional Standards for Teaching Mathematics* as an umbrella for the development of ideas over the course of all of the seminars. The mentor teachers are paid a small stipend, but it does not come close to compensating them for the richness they bring to Project MATH. When they are not leading seminar discussions, the teachers are an active part of the group, interacting with the undergraduates, doing the activities and exemplifying that learning is life-long; it does not stop with graduation. (See Appendix B for a mathematical problem that focuses on different ways of seeing and justifying a simple algebraic expression. See Appendix C for an activity which focuses on diagnosing common algebraic errors where the mentor teachers share the techniques that they use to help ensure that their students do not make these typical errors.)

In addition, Project MATH students have field experiences in the mentor teachers' classrooms starting as early as their freshman year. They visit multiple classrooms at both the middle and high schools with an assignment that asks them to observe specific things such as questioning techniques, including wait time. They also conduct scripted interviews with individual students trying to assess a student's understanding of a certain mathematical idea. They do individual tutoring in the classrooms with a culminating short five-minute mathematical presentation at the end of their first year. The activities grow each year so that by the end of their undergraduate career, they are extremely familiar and comfortable with the secondary classroom. The debriefing of many of these activities becomes the focus of the evening seminars.

As seniors, Project MATH students participate in the department's undergraduate teaching internship program discussed above. These students are in a position to become teacher leaders for their fellow interns and for their future colleagues in the secondary school.

<u>Secondary Partnerships: California Academic Performance Initiatives</u>

The California State University system has offered funding to the mathematics and English departments of the CSU to form partnerships with area secondary schools within their service regions. The purpose of these partnerships is to help reduce the need for remediation at the collegiate level as well as increasing the number of students who are qualified for higher education.

For the past three years California State University, Chico has been involved in mathematics partnership activities that encourage interactions between university faculty and their counterparts at the secondary school. Four main programs have evolved: a tutor program, ELM prep workshops, curriculum analysis, and math chat seminars.

The tutoring program is patterned after the tutoring described at the elementary level; the only difference is that these tutors are prospective secondary mathematics teachers and the tutoring targets Algebra 1 students. The decision to offer tutoring only at this level is two-fold. Algebra 1 is a critical course in the mathematics career of many students. If calculus is accused of being a filter rather than a pump at the college level, Algebra 1 shares this same role at the secondary level. The other reason is to ensure that the bi-weekly seminar has a thread of continuity and goes further than surface treatment of any one topic. A secondary mathematics teacher and two of our faculty jointly conduct the seminars.

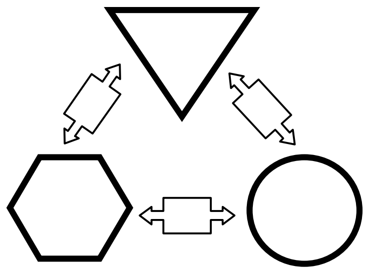
The Entry Level Mathematics (ELM) test workshops are a series of three workshops for secondary students to better prepare them for taking the ELM test. The workshops are conducted by a team of secondary teachers in conjunction with one of our faculty. The preparation for workshops provides a rich opportunity for teachers from both levels to communicate and discuss different mathematics topics.

Curriculum analysis is another component of this collaboration. All students in the secondary mathematics classes at the involved schools are given an appropriate form of the Mathematics Diagnostic Testing Project (MDTP) tests that are widely used throughout California. Teachers meet on a series of Saturday mornings with two university professors to look at the results from two different perspectives: examination of the relevant MDTP test for alignment with the California State Mathematics Standards and student performance on selected state standards across course levels. Working from the alignments obtained at the first meetings of the groups, after studying student performance on MDTP instruments and investigating closely CPM curriculum materials, groups focused on the areas of concern.

The math chat seminars are an opportunity to have two groups, university faculty and secondary teachers, informally get together and use mathematics as a platform for conversation. The topics have been varied and different in focus and goals. At times we have engaged in looking at an area of mathematics, e.g. cryptography or reform calculus, that is of interest to the audience, finding the fallacies in mathematical “proofs,” and discussing mathematical pedagogy through looking at case studies. The informal nature of these seminars has lead to more understanding and communication between the two groups.

Appendix A: Algebra Networks

Algebra networks are described in “Becoming Very-Able with Variables: Addition Using Algebra Networks” in the “Investigations” department of the February 1997 issue of *Teaching Children Mathematics*. See the figure below. Typically, the teacher presents the problem to the students by placing beans or some other kind of counter in the triangle, circle, and hexagon. Students are then asked to write the sum of the number of beans in the circle and the triangle in the rectangular space between these two figures. The same is done for the sum of the number of beans in the circle and the hexagon and in the hexagon and the triangle. When everyone agrees on the sums, the students remove the beans and, with just the sums remaining, the students attempt to determine the original number of beans in each of the three figures. Of course, in this example, students know there *is* a solution. But what if we randomly write numbers in the three rectangles? Can we be certain there is a solution? Could there be more than one solution?



The algebra network problem appeals to a wide variety of students. This is especially significant in our courses for future elementary teachers. Algebra networks are a standard topic in our upper division course in algebraic thinking. With the assistance of our Teacher-in-Residence, we have been able to introduce the problem to our university students while an elementary teacher simultaneously introduces the problem to fifth graders. After introducing the problem and providing time to share ideas about how to approach such problems, we ask the university students to create an appropriate problem for the elementary students to solve. By the next class meeting we are able to return the elementary school students' solutions and reasoning to the university students. The university students then analyze and critique the reasoning of the elementary students. The typical outcome of this activity is a profound sense of uneasiness on the part of the university students. The solution paths of the elementary students are often much more sophisticated than the preservice teachers expect; in fact, elementary students often discover techniques we have not discussed in our university classroom.

For the university students, algebra networks become one of three topics they may choose for a required course project. As a class, students brainstorm ideas about how to extend the algebra network problem. For example, some students have already discovered that the given algebra network configuration always yields a unique solution; these students often wonder if other configurations also yield unique solutions. For example, if four shapes are arranged in a square is the solution always unique? What if four shapes are arranged in a square and a diagonal is included? What if there are five shapes? Of course, there are many other directions the investigations might take. For example, students often ask questions that are variations on the following theme: What conditions will ensure that solutions are positive integers?

Appendix B: Swimming Pool Problem

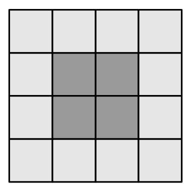
This is a well-known problem that can be used at many different levels. It is a wonderful problem environment that engages elementary children in rich algebraic thinking. (See "Experiences with Patterning" by J. Ferrini-Mundy, G. Lappan and E.

Phillips in *Teaching Children Mathematics,* February 1997.) We include the original problem environment here to set the stage for Part 2 which we use for secondary students and teachers.

Take **two** colors of tiles; so we can all talk about the water and the pool decking mathematically and not refer to the specific color of tiles. Let's agree that:

Dark Tile = Pool Water
Light Tile = Pool Decking

Tat Ming is designing square swimming pools. Each pool has a square center that is the area of the water. Use the Dark Tile to represent the pool water. Around the water, there is a pool deck that forms a border and goes entirely around the pool. Use the Light Tile to represent the pool deck.



A 2 x 2 pool

Let's generate some questions:

1. How many tiles are there altogether for each pool?
2. How many dark tiles? How many light tiles?
3. Are there more dark tiles than light tiles? Always? Never?
4. What patterns do you see in the dark tiles? In the light tiles? In the total number of tiles?
5. Can you predict how many tiles there will be in the next pool size?
6. Given the number of Pool Water tiles, can you determine the number of Pool Deck tiles? Vice-versa.
7. What numbers work for the number of Dark Tiles?
8. What numbers work for the number of Light Tiles?
9. What is the fractional relationship between the Dark Tiles and the Light Tiles used for each pool size? Between Dark Tiles and Total Tiles?
10. Can you tell me what 'pool 11' looks like?
11. Make a graph of the number of Dark Tiles used versus the pool size.
12. Make a graph of the number of Light Tiles used versus the pool size.
13. When do the Dark Tiles overtake the Light Tiles needed?

<u>Swimming Pool Problem (Part Two)</u>

Here is a sample of actual work from different groups of students working on the formula for the number of tiles in the border of the general case of a square pool that measures *x* by *x*. In your group, discuss these solutions and try to determine what might have been the thinking behind each of these formulas.

1. $1 + x + 1 + x + 1 + x + 1 + x$
2. $4(x + 1)$
3. $x + x + x + x + 4$
4. $(x + 2)^2 - x^2$
5. $4x + 4$
6. $4(x - 1) + 8$
7. $2(x + 2) + 2x$
8. $2(2x + 1 + 1)$
9. $2\left(x+2\left(\frac{x}{2}\right)+2\right)$
10. $2(2x+2)$
11. $2\left(2x+2\left(\frac{1}{2}\right)+1\right)$
12. $4(x+2)-4$
13. $(x+2)+2(x+1)+x$

Appendix C: Analyzing Algebra Errors
Project MATH Seminar Sample Discussion Items

For each of the following problems, first determine the correct solution. Then discuss what mathematical error(s) might have led to each of the incorrect "solutions." After identifying a possible source of the error, discuss ways that you, as a teacher, might help a student develop a deeper understanding so that s/he might avoid making a similar error in the future.

1. Expand the following: $(a-b)^2$

   a) a^2+b^2 b) a^2-b^2 c) $a^2+2ab+b^2$ d) $a^2-2ab+b^2$

2. Simplify the following: $\dfrac{(ab)^2}{a^2b^2c}$

   a) c b) $\dfrac{1}{c}$ c) $\dfrac{1}{ac}$ d) $\dfrac{c}{a}$

3. Solve for x: $3(x+2)-2(2x-3)=13$

   a) $x=-13$ b) $x=-1$ c) $x=25$ d) $x=-14$

4. Solve for x: $\sqrt{x^2+9}=5$

a) $x=2$ b) $x=-4$ c) $x=4$ d) $x=-4$ or $x=4$

5. $(3x+2y)^2=$

a) $9x^2+4y^2$ b) $9x^2+5xy+4y^2$
c) $9x^2+12xy+4y^2$ d) $9x^2+12xy+2y^2$

6. If $\begin{cases} x+y=2 \\ x-y=6 \end{cases}$, then $y=$

a) no solution b) 4 c) 8 d) -2

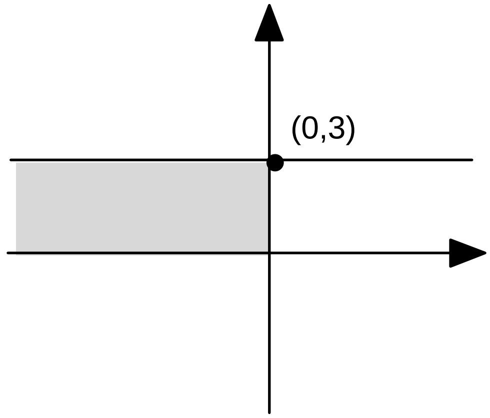
7. $4x+5\le 2x-3$ is equivalent to

a) $x\le -4$ b) $x\le 1$ c) $x\ge -4$ d) $x\ge 4$

8. $\dfrac{x^{a+1}}{x^2}=$

a) x^{a-1} b) $x^{\frac{a+1}{2}}$ c) x^{a+3} d) x^{2a+2}

9. Which of the following pairs of inequalities could describe the shaded region shown in the figure below?



a) $0 \le x \le 3$ and $y \ge 0$
b) $0 \le x \le 3$ and $0 \le y \le 3$
c) $x \ge 0$ and $0 \le y \le 3$
d) $x \le 0$ and $0 \le y \le 3$