Numbers and Operations in Base Ten, Grades K - 5

The work of teaching Numbers and Operations in Base Ten.

These standards concern representing, comparing, and calculating with numbers in base ten. The base-ten place value system provides for the remarkably efficient representation of numbers and for efficient, compact methods of calculation and of comparison. But this efficiency and compactness hides the underlying meaning, so teachers must explicitly bring meaning to the fore. The standards make clear that students are expected to understand the meaning of numbers represented in base ten and the rationale and logic underlying calculation methods. Much is known about how children can learn calculation methods with understanding, about the difficulties that must be surmounted and the common errors that occur, and about representations (such as drawings to show tens and ones) that can help children reason about and make sense of base-ten calculation methods.

At each grade, teachers help students make progress in their understanding of how the base ten system uses units of ten. For young children, understanding the whole numbers 11 through 19 as standing for 1 unit of ten and some ones is a major advance. Even older children may treat numbers in base ten as concatenations of digits rather than as involving units of ones, tens, hundreds, etc., unless they have strong instruction that highlights these base-ten units and the meaning of numbers written in base ten.

Efficient base-ten calculation methods for addition, subtraction, multiplication, and division rely on breaking numbers into their base-ten place value components and applying properties of arithmetic, including the commutative and associative properties of addition (often informally) and the distributive property, to decompose the calculation into parts. To help students learn calculation methods and make sense of the calculation methods they use, teachers will lead discussions with their class about the meaning and rationale of calculation methods. They help students learn to use and explain efficient methods with understanding, culminating in fluency.

Because of the uniform structure of the base-ten system (the value of each place is 10 times the value of the place to its right), comparisons (more/less than) and calculations with decimals follow the same logic as comparison and calculations with whole numbers. Teachers help students understand this common structure and use it in calculations, including understanding the reasoning behind the placement of the decimal point in calculations with decimals.

Key understandings to support this work

- Understand how the base ten system uses place value and bundling in units of ten to represent numbers. Use visual supports such as drawings, objects, and cards to show base-ten structure. Recognize that the way we say numbers in English does not always reveal base-ten structure.

- Understand the rationale and logic underlying efficient base-ten calculation methods (algorithms) and know ways to reveal and discuss the underlying rationale and logic with the aid of visual representations that show the quantities involved.
- Understand and write equations to show how properties of arithmetic, such as the distributive property, are used in justifying efficient base-ten calculation methods (algorithms). Know how to coordinate the equations with visual representations.
- Examine and evaluate hypothetical or actual student calculation methods to determine if they are mathematically valid or if they could be modified to become valid.
- Explain how to extend the base ten system to decimals and how to represent decimals on number lines.
- Know ways of explaining the rationale for placement of the decimal point in calculations with decimals.
- Know some common base-ten calculation errors, such as errors involving zeros that arise in division and in subtraction and errors that arise in interpreting and comparing decimals.

Illustrative examples

Before their coursework, prospective elementary teachers often do not realize that the common calculation algorithms can be explained in terms of more fundamental ideas. In other words, the calculation algorithms are frequently taken as "given." Considerable course time should be devoted to a careful development of the algorithms.

Prospective teachers can examine hypothetical or actual student calculation methods and decide if the methods are valid or not. For example, if a student calculates 23 x 45 by calculating 20 x 40 and 3 x 5 and adding the two results, is that method legitimate or not, why or why not, and if not, how could it be modified to become correct?

Problem: Examine this proposed method for rounding a number to the nearest hundred. Is it valid? 2367 => 2370 => 2400