AGENDA

- Introductions.
- A short history of our community.
- Stories from the members.
- How and why it has evolved.
- What has been especially difficult.
- What are the issues that you face in your work?
- Table and group discussion.
FOCUS ON MATHEMATICS

A Short History
Focus on Mathematics (FoM) is devoted to improving student achievement through programs that provide teachers with solid content-based professional development sustained by “mathematical learning communities” in which mathematicians, educators, administrators, and teachers work together to put mathematics at the core of 5–12 mathematics education.
A very brief “history” of FoM

- 1989: PROMYS (immersion experience for students)
- 1991: PROMYS for Teachers
- 1999: Academic year workshops added to PfT
- 2001: PCMI mathematics content course
- 2003: Focus on Mathematics
  - Study groups
  - Masters program (Mathematics for Teaching)
  - Teacher-led institutes and seminars
- 2009: FoM Phase II (research program)
PfT is a professional development program for secondary mathematics teachers, consisting of the following components:

- A six-week summer “immersion experience” in mathematics (elementary number theory).

- Second summer “shadow seminar” to analyze the immersion experience from a pedagogical point of view.

- Academic year workshops that connect the immersion experience to teachers’ work in the classroom.
IMMERSION EXPERIENCE
(1991 – present)

Teachers and mathematicians doing mathematics
○ as an empirical science ("Experience First")
○ as exploration
○ alongside high school students
○ as a community activity

Key Features:
○ quality and fidelity to mathematics
○ strengthening mathematical habits of mind
○ low threshold, high ceiling
○ deeply personal engagement with mathematics
A typical day at PfT
The immersion experience is supported and enriched by the PfT counselors who

- help teachers *struggle productively*,
- model the pedagogy of “questioning answers,”
- provide daily written & verbal feedback on problem sets,
- meet weekly to discuss the progress of each teacher.

**Note:** Half of the PfT counselors are former participants.
PfT Second Summer Shadow Seminar

This seminar is for teachers returning to PfT for the second summer. The goal of the seminar is to explore

- the ways in which secondary teachers know and use mathematics in their profession,

- how to bring the pedagogical philosophy and style of PROMYS into the secondary mathematics curriculum, and

- the effects that such a learning environment might have on secondary students.
A C A D E M I C  Y E A R  W O R K S H O P S  
(1999 – present)

Connecting the immersion to the classroom
- Teachers sharing classroom experiences & activities
- Discussion of curriculum and education policy
- More (and more) mathematics
- Continued development of the mathematical community

Key Features:
- quality and fidelity to mathematics
- strengthen students’ mathematical habits of mind
- low threshold, high ceiling
- ongoing engagement with mathematics
Our motto:

*Building a Community of Mathematicians, Teachers, and Educators.*

Our approach:

- Depth over breadth
- Focus on mathematics
- Capacity building
  - Teachers learn to drive professional development.
- Community building
The MMT targets experienced teachers who have a strong commitment to teaching, leadership, and a love of mathematics. The program prepares these individuals for leadership roles in developing curriculum and training other teachers.

Elements of the program

- an immersion experience (i.e., PROMYS)
- a mathematical research experience
- leadership experiences
SAMPLE RESEARCH PROJECTS

- Patterns in Pascal’s triangles
- Repeating decimals and other bases
- Sums of Squares
- Pythagorean Triples
- Combinations and Partitions
- Dynamics of billiards on a circular table
- Stirling Numbers of the second kind
- Symmetries of cubes in higher dimensions
- Applications of quaternions to geometry
Duplicates in the $n \times n$ Lattice

- There are other duplicates in the $n \times n$ lattice that are not of the form $(a,b)$ and $(b,a)$.
- For example:
  \[ 0^2 + 5^2 - 3^2 + 4^2 = 5^2 = 25 \]
- These distances can be thought of as norms in the complex plane.
- $N(5i) = N(3 + 4i) = 25$
It is the best “professional development” that I have been involved in throughout my 35-year teaching career. I guess the best testament for the success of Focus on Mathematics comes from the continued attendance of so many teachers. We continue to talk about the topics discussed at our study groups long after the weekly session is over.

– FoM teacher
FoM Study Groups

Our approach

- Mathematicians working with teachers as colleagues
- Sharing expertise
- Connecting to mathematics for teaching
- Increasing active involvement by teachers
- Teacher-led sessions
Some details (from LHS group)

- Currently meet biweekly throughout the school year.
- Average attendance is about 10-12 teachers with nearly half of the math teachers participating regularly.
- Al Cuoco has been with us from the start and we have also worked with Bob Devaney and Marvin Freedman from BU and Ken Levasseur, Marvin Stick and Kiwi Graham-Eagle from UM-Lowell.
FoM Study Groups

- First, we have fun doing math together.
- Sometimes we work on problems that connect to the mathematics we teach in our classes.
- Sometimes we just work on what seems interesting.
- We always learn something.
- It makes us think about ways to give our students similar mathematical experiences.
Abstract: Are triangles with the same area and perimeter congruent? The authors relate this seemingly simple question to Heron’s Formula, Arithmetic-Geometric Mean Inequality, Calculus and Number Theory.
**FoM Summer Institutes**

- Week-long courses offering teachers an immersion experience in mathematics.
- Features similar to those of the PfT immersion experience (i.e., experience first, low threshold/high ceiling, MHoM).
- Developed and facilitated by teachers in recent years.
- Teachers have the opportunity and time to work with their colleagues, thus forming a strong, connected community.
Sample Institute Topics

- Algebra Connections
- Problem Solving and Modeling with Graph Theory
- Problem Solving in Number Theory and Algebra
- Algebraic Reasoning
- Exploring Topics in Statistics, Probability, and Finance
- Fostering Geometric Thinking
Our work with secondary teachers has led us to believe that when teachers engage deeply with mathematics, there is a positive effect on their classroom instruction.

Recognizing the need for a scientific approach to validate our anecdotal evidences, we began to conduct a research study centering on the following question:

What are the mathematical habits of mind that high school teachers use in their professional lives and how can we measure them?
We are currently working to identify and precisely define MHoM, and to operationalize this framework into paper and pencil assessment problems that accurately and uniquely measure these habits.

This study is a first step in our proposed long-term research program with the goal of understanding the connections between secondary teachers’ MKT and their students’ learning and achievement.
The MMT program has expanded my knowledge of mathematics and deepened my understanding of how children learn mathematics, but – more importantly – I am now connected to people who are as passionate about children learning and doing mathematics as I am.

– FoM teacher
CHALLENGES AND OPPORTUNITIES

- The *necessity versus sufficiency* problem
- Establishing and sustaining coherent goals
- Bringing it to the classroom
  - Beliefs about students
  - Beliefs about mathematics
- Resources – money and people
- Building and sustaining strong communities of teachers
  - Beliefs about teachers
  - Grassroots organizing in the schools
  - Administrators enabling teacher communities – autonomy, discipline-based professional development.
- Where are all the mathematicians?
  - Mathematician involvement
  - Teachers as mathematicians
THANK YOU

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