Chapter 1 footnotes with hyperlinks

The footnotes from Chapter 1 are listed below and hyperlinked (when possible) to the references cited.

Many of the documents cited are freely available. National Research Council reports such as *Adding It Up* can be read on-line. They can be downloaded without charge as can documents from the Conference Board of the Mathematical Sciences and the Council of Chief State School Officers. In some cases, cited portions of documents can be seen via Google Books.

Mathematics education research journal articles are likely to require a subscription. At many academic institutions, these journals will be accessible via institutional subscription. Attempts to access a JSTOR link without such a subscription will get the response "Cannot download the information you requested."

1. An overview of the CCSS structure appears as Appendix B of this report.

2. The full text of these standards appears as Appendix C.

3. Between 2004 and 2008, the Park City Mathematics Study Group (a group of research mathematicians) conducted discussions of school mathematics, including extended discussions with NCTM representatives. *Principles and Standards* and *Adding It Up* (published in 2000 and 2001) summarize findings from previous decades of research in mathematics education.

4. Such connections are outlined in the Progressions for the CCSS (see the web resources for this report).

5. Examples are given by Ma, *Knowing and Teaching Elementary Mathematics*, Erlbaum, 1999: change in number, p. 74; change in manipulative and problem context, p. 5.

6. For a summary (p. 400) and further examples of teaching tasks, see Ball et al., "Content Knowledge for Teaching," *Journal of Teacher Education*, 2008; also Senk et al., "Knowledge of Future Primary Teachers for Teaching Mathematics: An International Comparative Study," *ZDM*, 2012, p. 310.

7. See, e.g., the findings of the Teacher Education and Development Study in Mathematics (TEDS-M).

8. These are intertwined and occur on a variety of levels. For example, the institutional arrangement of having teachers share a room affords the professional practice of discussing mathematics. An institutionalized career hierarchy based on teaching shapes the professional activities of Chinese master teachers and "super rank" teachers described in *The Teacher Development Continuum in the United States and China*, National Academies Press, 2010. In Japan, institutional arrangements afford the practice of "lesson study," allowing teachers to communicate with other teachers in their

school or district, and with policy-makers (see Lewis, *Lesson Study*, Research for Better Schools, 2002, pp. 20–22).

9. Chapter 2 discusses this claim further, but note the findings of *Effects of Teacher Professional Development on Gains in Student Achievement*, Council of Chief State School Officers, 2009. Most successful professional development programs continued for 6 months or more, and the mean contact time with teachers was 91 hours.

10. For example, the Mathematics Common Core Coalition (comprised of professional societies and assessment consortia) addresses educators, teachers, teacher leaders, supervisors, administrators, governors and their staffs, other policy-makers, and parents.

11. The CBMS surveys (conducted every five years) consistently document large proportions of undergraduates enrolled in remedial mathematics courses (see, e.g., Table S.2 of the 2005 report).

12. The 2005 CBMS survey suggests that many mathematics departments do not have courses especially designed for elementary teachers (see Table SP.6). In 2010, Masingila et al. surveyed 1,926 U.S. higher education institutions that prepared elementary teachers. Of those who responded (43%), about half (54%) reported that requirements included two mathematics courses designed for teachers. See "Who Teaches Mathematics Content Courses for Prospective Elementary Teachers in the United States? Results of a National Survey," *Journal of Mathematics Teacher Education*, 2012, Table 2. A more detailed picture for three states is presented by McCrory & Cannata, "Mathematics Classes for Future Elementary Teachers: Data from Mathematics Departments," *Notices of the American Mathematical Society*, 2011.

13. Chapter 2 gives an overview of teaching–learning paths.

14. In Masingila et al.'s survey less than half of respondents reported giving training or support to instructors of mathematics courses for elementary teachers.

15. For example, when surveyed in 2000, 86% of K–4 teachers reported studying mathematics for less than 35 hours over a period of three years, an average of less than 12 hours per year. See Horizon Research's 2000 National Survey of Science and Mathematics Education. More recent studies show large increases in elementary student mathematics achievement when their teachers receive content-based professional development. Student scores of teachers who do not receive such professional development do not show these gains (see the sections on curriculum-specific professional development in Chapter 2 and on mathematics specialists in Chapter 4). Thus, unsatisfactory student performance may suggest a greater need for content-based professional development.

16. The Association for Middle Level Education (AMLE) position statement notes, "in some states,

virtually anyone with any kind of degree or licensure is permitted to teach young adolescents." According to the AMLE web site, 28 states and the District of Columbia offer separate licenses for middle grades generalists. Separate licenses, however, do not necessarily imply the existence of separate preparation programs or different mathematics requirements. The 2005 CBMS survey found that 56% of mathematics departments at four-year institutions had the same mathematics requirements for K–8 certification in early and later grades (see Table SP.5). See also the discussion of opportunity to learn for U.S. prospective lower secondary teachers in Tatto & Senk, "The Mathematics Education of Future Primary and Secondary Teachers: Methods and Findings from the Teacher Education and Development Study in Mathematics," Journal of Mathematics Teacher Education, 2011, p. 127.