Chapter 5 footnotes with hyperlinks

The footnotes from Chapter 5 are listed below and hyperlinked (when possible) to the references cited.

Many of the documents cited are freely available. National Research Council reports such as *Adding It Up* can be read on-line. They can be downloaded without charge as can documents from the Conference Board of the Mathematical Sciences and the Council of Chief State School Officers. In some cases, cited portions of documents can be seen via Google Books.

Mathematics education research journal articles are likely to require a subscription. At many academic institutions, these journals will be accessible via institutional subscription. Attempts to access a JSTOR link without such a subscription will get the response "Cannot download the information you requested."

Note that the MET II web resources at www.cbmsweb.org give URLs for the CCSS, the Progressions for the CCSS, and other relevant information.

1. As noted in Chapter 3, "Although elementary certification in most states is still a K–6 and, in some states, a K–8 certification, state education departments and accreditation associations are urged to require all grade 5–8 teachers of mathematics to satisfy the 24-hour requirement recommended by this report." Chapters 4 and 5 allow for a period of transition.

2. See the listing at the Association for Middle Level Education web site.

3. See, e.g., Tatto & Senk, "The Mathematics Education of Future Primary and Secondary Teachers: Methods and Findings from the Teacher Education and Development Study in Mathematics," *Journal of Mathematics Teacher Education*, 2011, p. 127; *Report of the 2000 National Survey of Mathematics and Science Education*, Horizon Research, p. 16.

4. See the Ratio and Proportion Progression for further details, including examples of double number lines and tape diagrams, and discussion of unit rates. In the CCSS, "fractions" refers to non-negative rational numbers in grades 3–5. Note that distinctions made in the CCSS between fractions, ratios, and rates may be unfamiliar to teachers.

5. For descriptions of multiplication and division problem types, see the CCSS, p. 89 or the Operations and Algebraic Thinking Progression.

6. "I don't think it's equal because I think that would be confusing to kids to say that 99 cents can be rounded up to a dollar" and other examples of conceptions that teachers may hold about this equation are given in Yopp et al., "Why It is Important for In-service Elementary Mathematics Teachers to Understand the Equality .999. . . = 1," *Journal of Mathematical Behavior*, 2008. Note that undergraduates may use decimal notation in ways that suggest notions of nonstandard analysis, see Ely, "Nonstandard Student Conceptions About Infinitesimals," *Journal for Research in Mathematics Education*, 2010.

7. It is important for middle grades teachers to have an elementary teacher's perspective on this content because they may need to provide support and instruction for students who have not yet achieved proficiency.

8. Note that the CCSS use the term "number line diagram" instead of "number line."

9. Examples of these representations occur in the Progressions for the CCSS.

10. See the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A PreK–12 Curriculum Framework* of the American Statistical Association.

11. Lesson study is a process in which teachers jointly plan, observe, analyze, and refine actual classroom lessons.

12. Math teachers' circles and immersion experiences focus primarily on giving teachers an experience to be learners and doers of mathematics. See Chapter 6 and the web resources for further information and examples.

13. See, e.g., discussion of the use and organization of the blackboard in Lewis, *Lesson Study*, Research for Better Schools, 2002, pp. 97–98.

14. See Kidwell et al., *Tools for Teaching Mathematics in the United States, 1800–2000*, Johns Hopkins University Press, 2008.