Catalyzing Change in High School Mathematics: Initiating Critical Conversations

Panel Presentation

CBMS Meeting: May 4, 2018

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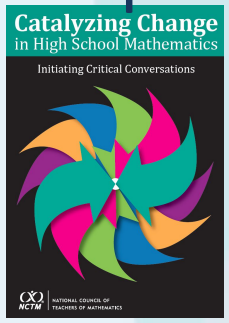
Outline

- Why *Catalyzing Change?*
- Overview of Key Recommendations
  - Broadening Purposes
  - Equitable Structures & Teaching
- Essential Concepts
  - Role of Modeling, Proof, Technology
- Organizing the High School Curriculum
  - What Counts as a Mathematics Course

Why *Catalyzing Change*?

- too many HS students develop unproductive mathematical identities and see little value in mathematics
- the need for mathematical skills is increasing for:
  - the workplace, postsecondary education requirements
  - active participation in our democratic society
- the HS math standards lack focus (too many)
- critical conversations are needed with all stakeholders to change policies and practices, not just content



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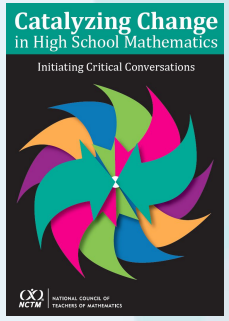
Key Recommendation

- *Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.*
- *High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.*



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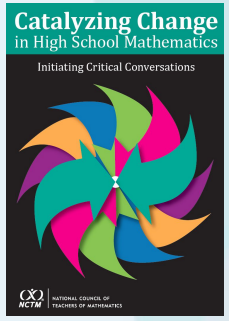
Key Recommendation

- *Classroom instruction should be consistent with research-informed and equitable teaching practices.*
- *High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts.*



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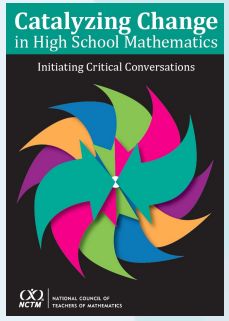
Broadening the Purposes of HS Math

- Note the focus beyond college/career readiness:
  - *Expand professional opportunities*
  - *Understand and critique the world*
  - *Experience the joy, wonder, and beauty of mathematics*

Making the multiple reasons for teaching and learning mathematics clear to both teachers and students can serve as a powerful way to increase student engagement with and motivation to learn mathematics.

Equitable Structures & Teaching Practices

- Eliminating Student & Teacher Tracking
- Distinguishing Tracking vs. Appropriate Acceleration
  - Acceleration should be along a single common shared pathway that provides each student with an opportunity to learn the same Essential Concepts



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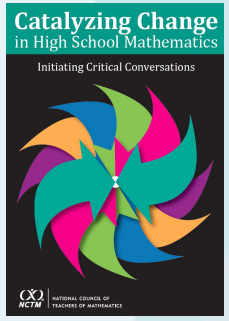
Equitable Structures & Teaching Practices

- Connecting Math Teaching Practices to Equitable Teaching

Mathematics Teaching Practices: Supporting Equitable Mathematics Teaching	
Mathematics Teaching Practices	Equitable Teaching
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	• Establish learning progressions that build students' mathematical understanding, increase their confidence, and support their mathematical identities as doers of mathematics. • Establish high expectations to ensure that each and every student has the opportunity to meet the mathematical goals. • Establish classroom norms for participation that position each and every student as a competent mathematics thinker. • Establish classroom environments that promote learning mathematics as just, equitable, and inclusive.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	• Engage students in tasks that provide multiple pathways for success and that require reasoning, problem solving, and modeling, thus enhancing each student's mathematical identity and sense of agency. • Engage students in tasks that are culturally relevant

Essential Concepts

- *Catalyzing Change* identifies a set of **Essential Concepts** from the content domains of number, algebra and functions, statistics and probability, and geometry and measurement.
- Essential Concepts represent the **most critical content from the content domains** – the deep understandings that are important for students to remember long after they have forgotten how to carry out specific techniques or apply particular formulas.



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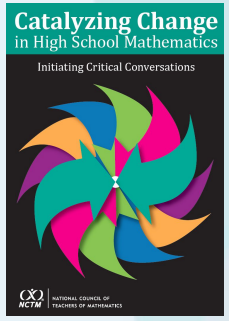
Essential Concepts

The Essential Concepts represent a distillation of the critical concepts and skills that, regardless of a state's, province's, or district's standards, students should acquire. Essential Concepts **do not represent yet another set of standards** or a list of disjoint topics to be covered.



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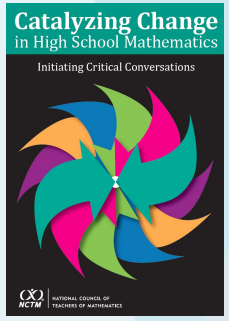
Content Area and Focus	Number of Essential Concepts
Number	2
Algebra and Functions	
Focus 1: Algebra	4
Focus 2: Connecting Algebra to Functions	2
Focus 3: Functions	4
Statistics and Probability	
Focus 1: Quantitative Literacy	2
Focus 2: Visualizing and Summarizing Data	6
Focus 3: Statistical Inference	7
Focus 4: Probability	2
Geometry and Measurement	
Focus 1: Measurement	3
Focus 2: Transformations	4
Focus 3: Geometric Arguments, Reasoning, and Proof	3
Focus 4: Solving Applied Problems and Modeling in Geometry	2

Essential Concepts in Algebra and Functions...

"The study of **algebra** and **functions** provides experiences for students to see how mathematics can be used systematically to represent patterns and relationships among numbers and other objects, analyze change, and model everyday events and problems of life and society."



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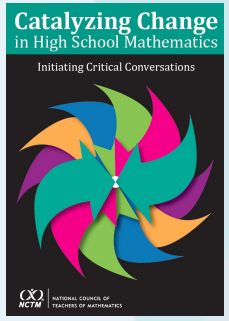
Essential Concepts in Algebra and Functions...

Example Essential Concepts:

- The structure of an equation or inequality… can be purposefully analyzed… to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.

- Expressions can be rewritten in equivalent forms by using algebraic properties... to make different characteristics or features visible.



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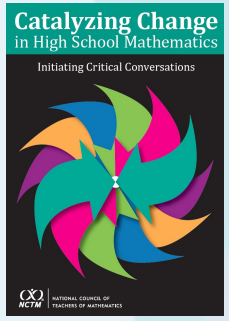
Essential Concepts in Algebra and Functions...

Example Essential Concepts:

- Functions shift the emphasis from a point-by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.



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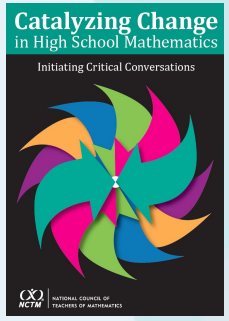
Essential Concepts in Statistics and Probability....

- "All high school graduates will, as members of society, be presented with data-based claims throughout their lives. Therefore, they must be able to examine these claims and be intelligent consumers of studies, capable of reasoning critically and asking questions about the implementation of the statistical investigation process in those studies."



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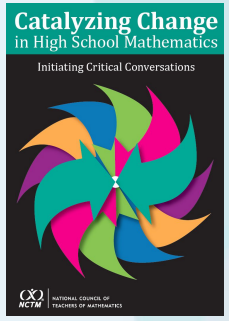
Essential Concepts in Statistics and Probability....

Example Essential Concepts:

- Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.

- The scope and validity of statistical inferences are dependent on the role of randomization in the study design.



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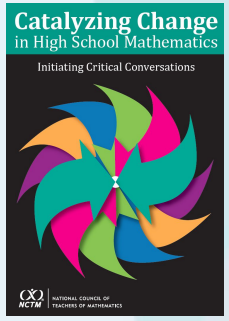
Essential Concepts in Statistics and Probability....

Example Essential Concepts:

- Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure—a first step in any analysis of data.



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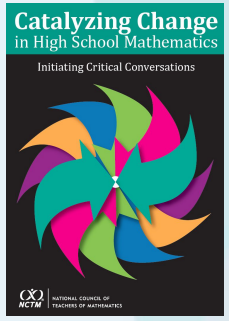
Essential Concepts in Geometry and Measurement...

Reflect three developments:

- Using transformations to study congruence, similarity, and symmetry
- Employing coordinates to enable connections between algebra and function and geometric ideas
- Taking advantage of dynamic graphics technology



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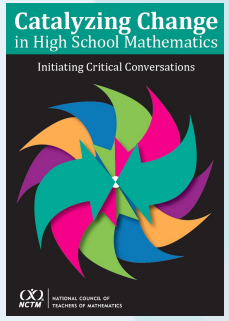
Essential Concepts in Geometry and Measurement...

Example Essential Concepts:

- Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself.



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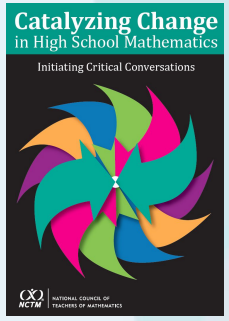
Essential Concepts in Geometry and Measurement...

Example Essential Concepts:

- Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases one may provide a more accessible or understandable argument than another.



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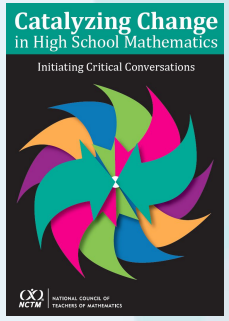
Essential Concepts in Geometry and Measurement...

Example Essential Concepts:

- Experiencing the mathematical modeling cycle in problems involving geometric concepts, from the simplification of the real problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility, introduces geometric techniques, tools, and points of view that are valuable to problem solving.



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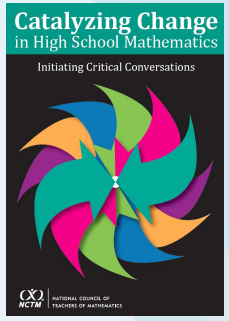
Role of Modeling, Proof, and Technology

Reasoning/Proof and Modeling:

- cut across content areas
- empower students
- support development of math identity and agency

Technology:

- to explore, build intuition, deepen understanding



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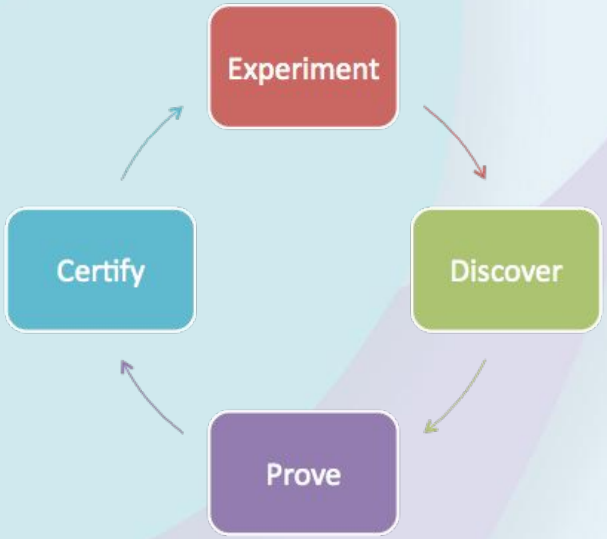
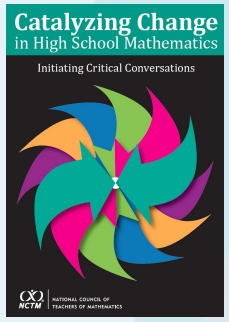
Role of Modeling, Proof, and Technology

Proof: Cycle of inquiry and justification

- Inquiry: exploration, discovery, conjecture
- Justification: proof, certification



Often, inquiry is ignored, but inquiry supports identity, agency, empowerment.



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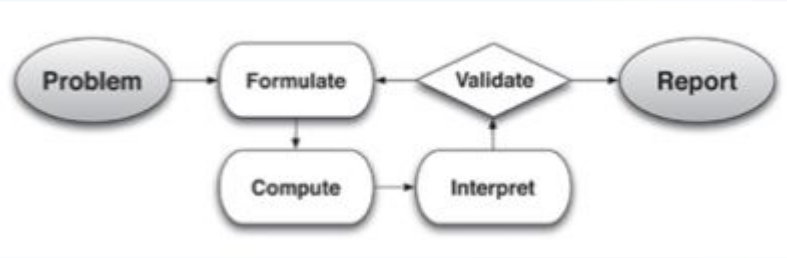
Role of Modeling, Proof, and Technology

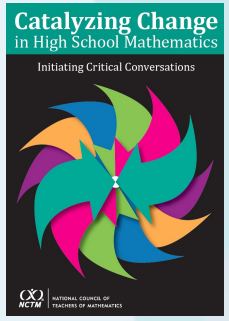
Modeling: often presented as a cycle

- posing questions
- translating to math, and solving
- interpreting solution
- refining the question

Modeling also supports identity, agency, empowerment.

IMMERSION





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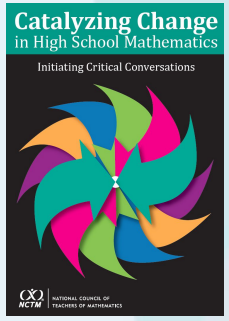
Key Recommendation

- High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts.



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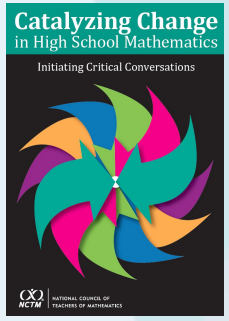
Sample Pathways

- Pathway A—Geometry First
- Pathway B—Integrated Approach
- Guard against the “race to calculus”
- Need for “targeted instructional support” for those beginning grade 9 without necessary foundation in K-8 mathematics curriculum



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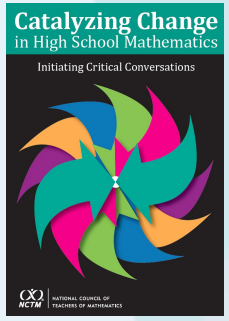
Pathway A—Geometry First
2 ½ Year

- **Grade 9**: Geometry and Measurement Essential Concepts followed by Statistics and Probability Essential Concepts
- **Grade 10:** Algebra and Functions Essential Concepts
- **First Half of Grade 11:** Integrate Algebra, Geometry, and Statistics with emphasis on practices and processes.



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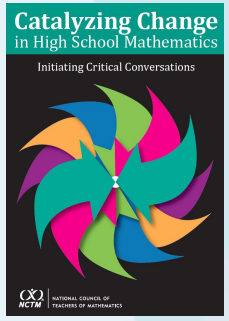
Pathway B—Integrated Approach
3 Year

- **Grade 9**: Integration of geometry and measurement with statistics and probability
- **Grade 10:** Algebra and functions
- **Grade 11:** Integration of functions, modeling and statistical inference



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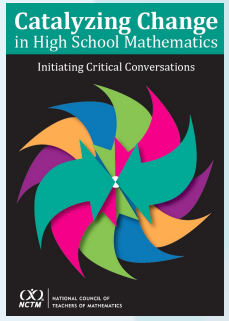
Possible **Course Options** beyond the **Essential Concepts**

- Precalculus
- AP Calculus
- AP Statistics
- IB Mathematical Studies
- Quantitative Literacy
- Financial Mathematics
- History of Mathematics
- Mathematical Modeling
- Discrete Mathematics

- Advanced Quantitative Reasoning (AQR) / Advanced Mathematical Decision Making (AMDM) (Dana Center 2017a)
- Statway and Quantway (Carnegie Math Pathways 2017a)
- Math Ready: Ready for College-Level Math (Southern Regional Education Board 2016)



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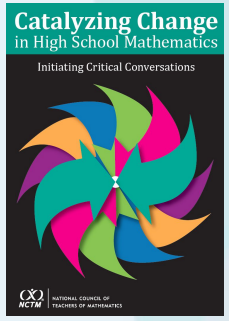
What Counts as a Mathematics Course?

Only courses that address mathematical standards (including statistics) and that are mathematically demanding should count toward high school mathematics graduation requirements.



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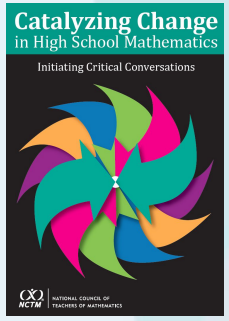
Mathematically Demanding Courses

- Require clarity and precision in reasoning
- Have focused and significant mathematics learning standards
- Maintain the integrity of the mathematical standards
- Are part of coherent mathematical learning progression (not dead-end courses)
- Approach mathematics in an instructionally balanced way



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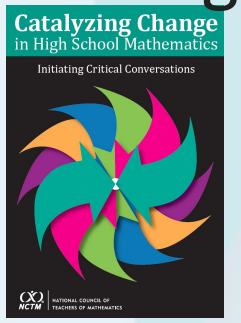
Discussion - What Do Students Need:

- to be an educated citizen
- to be prepared for a Pathways program at a post-secondary institution
- to be prepared for a technical program or a less math-intensive STEM trajectory
- to be prepared for a math-intensive STEM trajectory such as engineering, physical science, or mathematical science



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