CBMS: Deep Learning and Numerical PDEs

Conference Introduction:

The Conference Board of the Mathematical Sciences conference "Deep Learning and Numerical PDEs" will be held at the Department of Mathematics at Morgan State University. This conference begins June 19th, 2023, and ends June 23rd, 2023. This conference will bring together experts from mathematics, computer science, and engineering on topics involving deep learning and numerical partial differential equations (PDEs). The conference will provide good opportunities for students and junior researchers to acquire fundamental knowledge of deep learning. Furthermore, it will serve as a dynamic forum for disseminating the latest advancements in this rapidly evolving field, fostering enhanced collaborations among research groups from different universities. This conference consists of a series of ten lectures given by Prof. Jinchao Xu from KAUST and Penn State. His daily lectures will focus on the intersection of deep learning and numerical PDEs. Additionally, it will have several invited speakers who will discuss the latest advancements in these domains. More details on the conference website: https://sites.google.com/view/nsf-cbms-dl-nmpde/home.

Course Introduction:

This is a ten-lecture short course on the mathematical, numerical and practical aspects of deep learning techniques and their interaction with numerical PDEs. It will provide students with the mathematical background and practical tools needed to understand, analyze and further develop deep learning methods and applications from the numerical perspective. The short course is simultaneously geared towards math students and junior researchers who want to learn about the emerging technology of deep learning and also towards students from other fields who are interested in deep learning and numerical PDEs but would like to strengthen their theoretical foundation and mathematical understanding. This comprehensive short course serves as a bridge between the fields of Numerical PDEs and Deep Learning, covering a wide range of topics. It begins with an introduction to Numerical PDEs and Finite Element Method (FEM), providing a solid foundation. Subsequently, the course delves into subspace corrections and Multigrid (MG) methods, emphasizing their interconnectedness with network architecture and training of deep learning models. The exploration of approximation and representation theory of neural networks encompasses both fundamental principles and advanced concepts. Additionally, the course tackles training challenges by investigating frequency bias and greedy algorithms within the context of finite neuron methods for numerical PDEs and their error estimates. Lastly, MgNet, a comprehensive framework unifying Convolutional Neural Networks (CNNs) and multigrid methods, is introduced, highlighting its diverse applications across various domains.

Course Reading List:

1. DL course at KAUST and Penn State by Jinchao Xu, Website.
2. Goodfellow I., Bengio Y. and Courville A. Deep learning. MIT Press, 2016.
3. Xu J. Deep Learning and Analysis, Lecture Notes (to be published by Springer).
4. For related papers, see Google Scholar or Homepage.

Course Outline:

- Day 1: Finite element and ReLU DNNs
  - Introduction
  - FE spaces and ReLU^k neural networks
  - ReLU DNN = linear FE
  - ReLU DNN and hierarchical basis
- Day 2 (): Iterative methods and frequency principle
  - Basic iterative methods
  - Frequency principles
  - Multigrid methods
- Day 3: Image classification and MgNet
  - Logistic regression
  - Image classification
  - MgNet: A ``trained" multigrid method
- Day 4: Application to PDEs: Finite neuron method
  - Error analysis
  - Novel training algorithms
- Day 5: Approximation theory of neural network functions
  - Sampling argument
  - Variation space
  - Metric entropy